

# Toward a new generation of reflection models for precision measurements of accreting black holes

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**Fudan University**



NAOC, Beijing (18 October 2023)



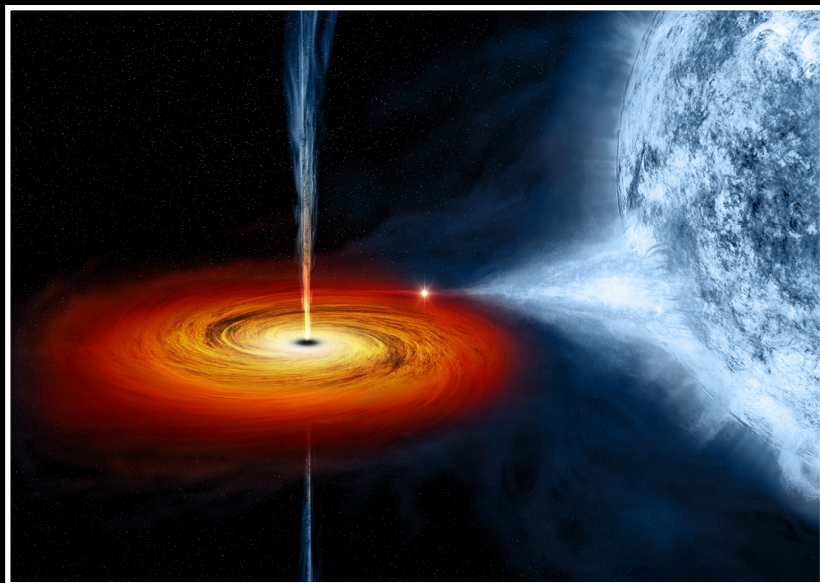
**Part I:**

# **X-Ray Reflection Spectroscopy**

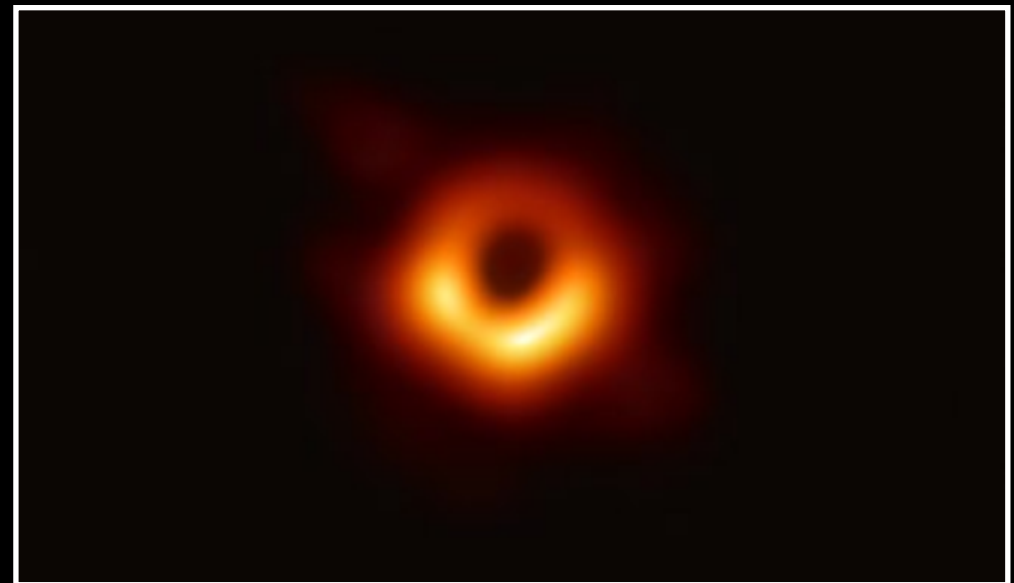
# Introduction

# Astrophysical Black Holes

Stellar-mass black holes  
( $\sim 3\text{-}100\ M_{\odot}$ )



Supermassive black holes  
( $\sim 10^5\text{-}10^{10}\ M_{\odot}$ )



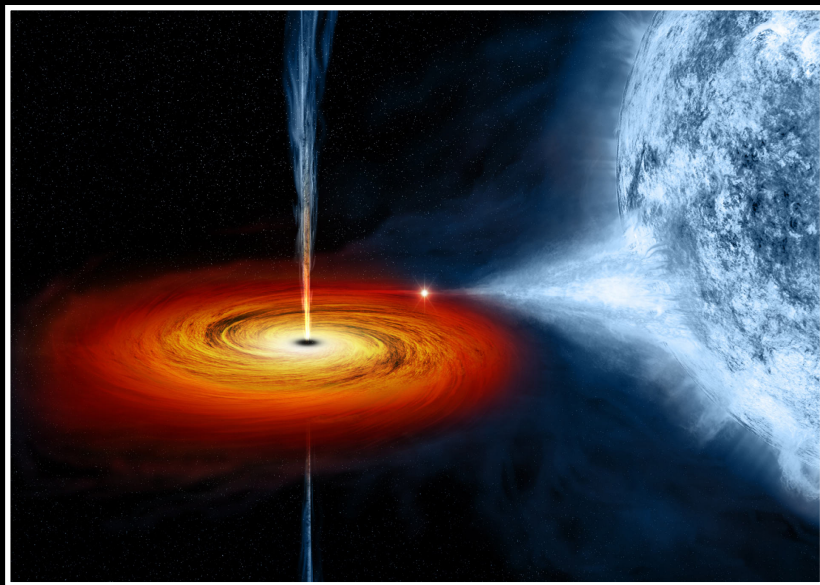
Intermediate-mass  
black holes





# Astrophysical Black Holes

Stellar-mass black holes  
( $\sim 3\text{-}100 M_{\odot}$ )



$\sim 10^8\text{-}10^9$  objects  
in the Milky Way

$\sim 70$  objects  
in X-ray binary systems

$\sim 40$  objects  
with a measurement  
of the spin with XRS

$\sim$  a few objects  
suitable for  
precise GR tests with XRS

# Astrophysical Black Holes

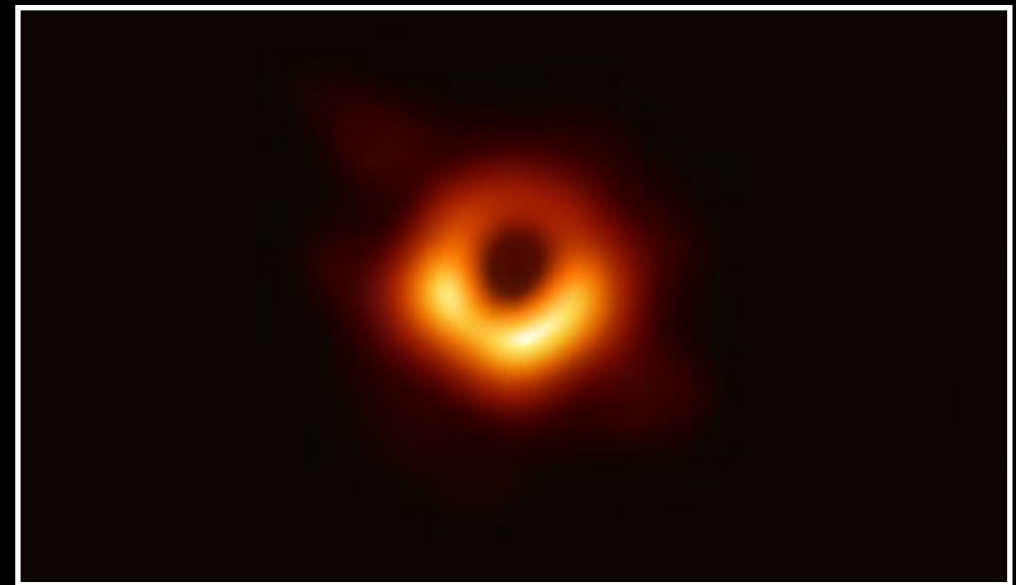
$\sim 10^{12}$  galaxies in the  
observable Universe

0.035% AGN

$\sim 40$  objects  
with a measurement  
of the spin with XRS

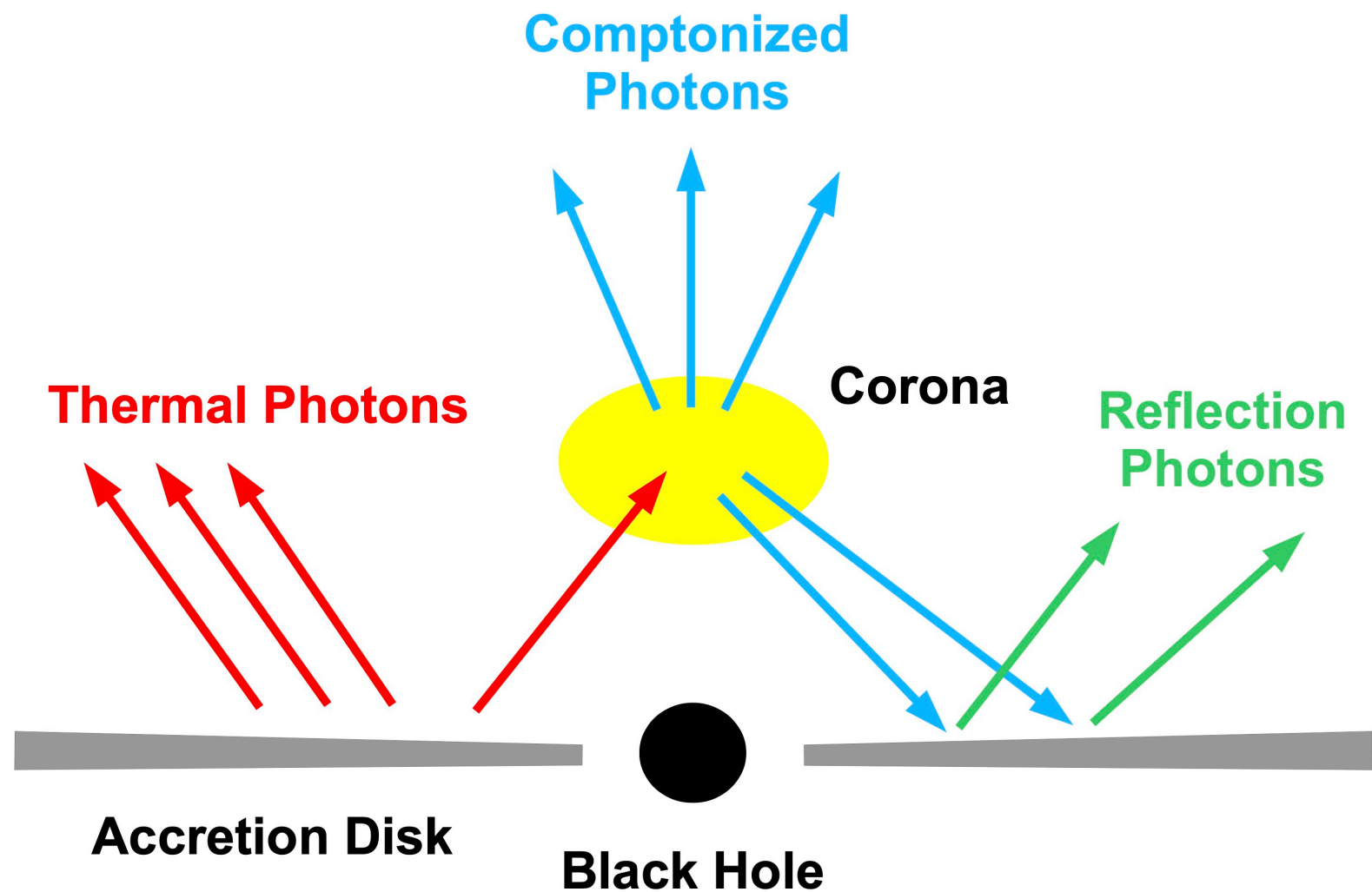
$\sim$  a few objects  
suitable for  
precise GR tests with XRS

Supermassive black holes  
( $\sim 10^5 - 10^{10} M_{\odot}$ )

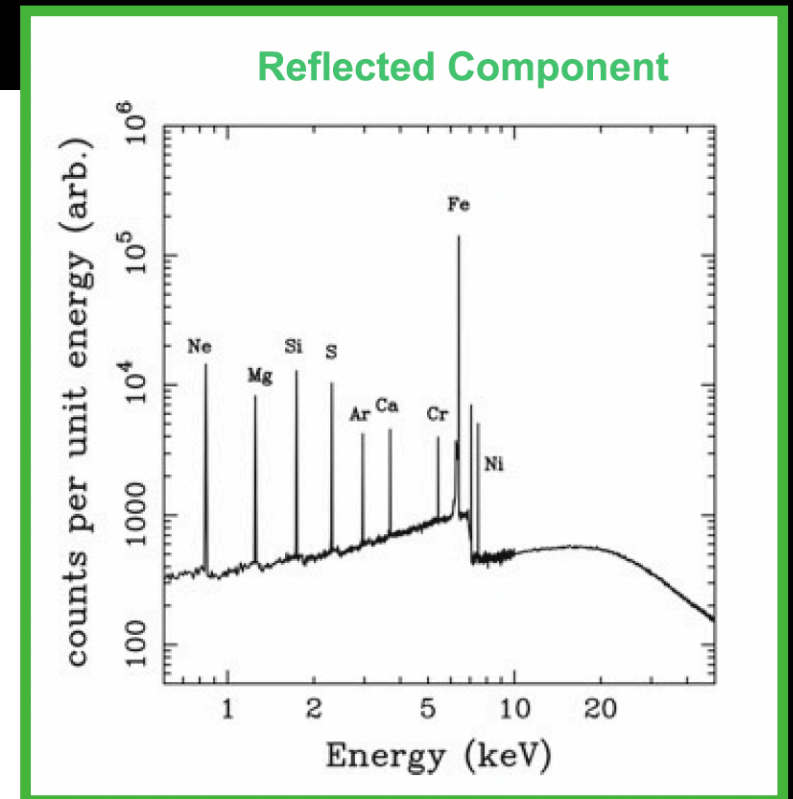
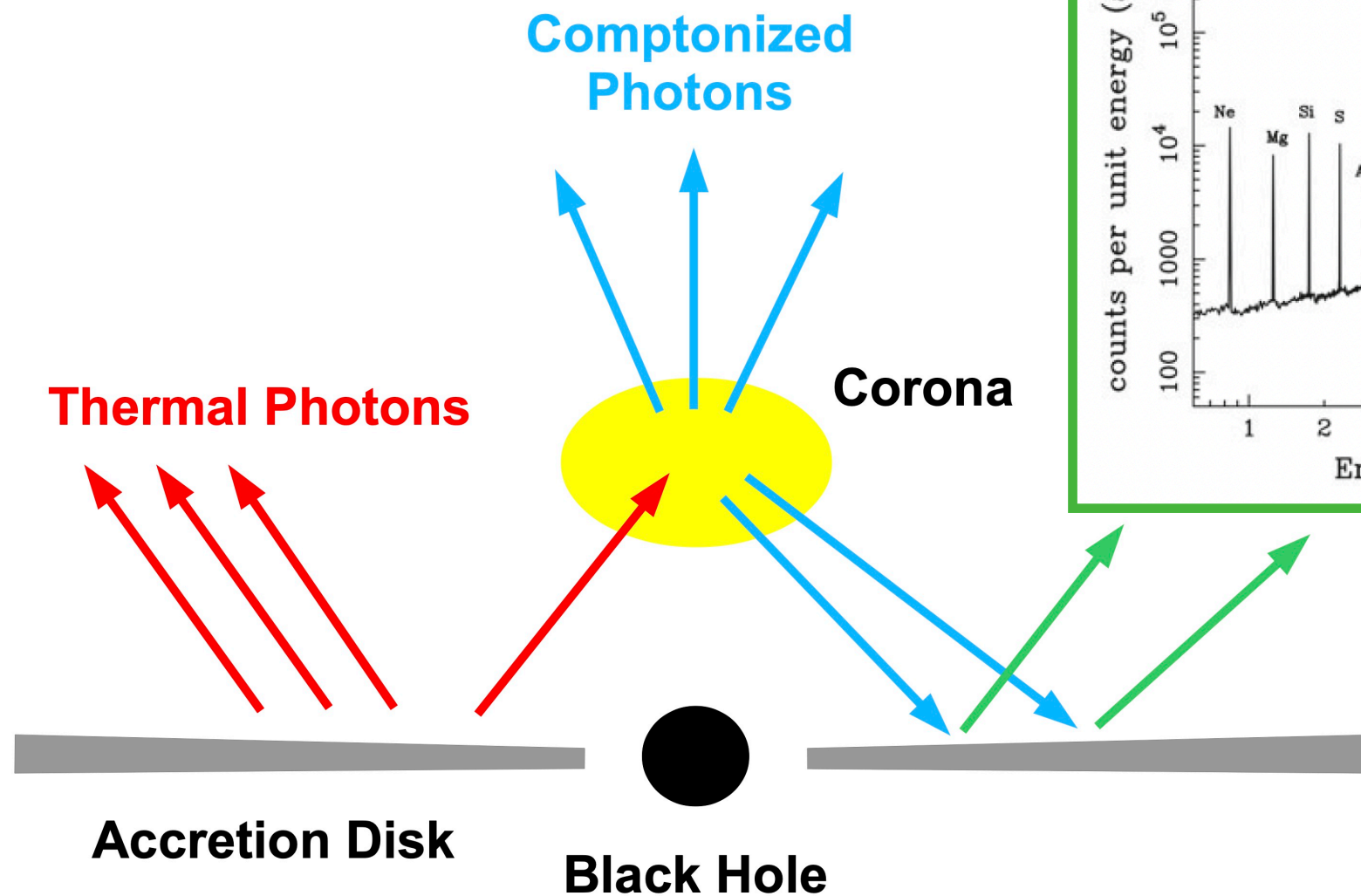


# Disk-Corona Model

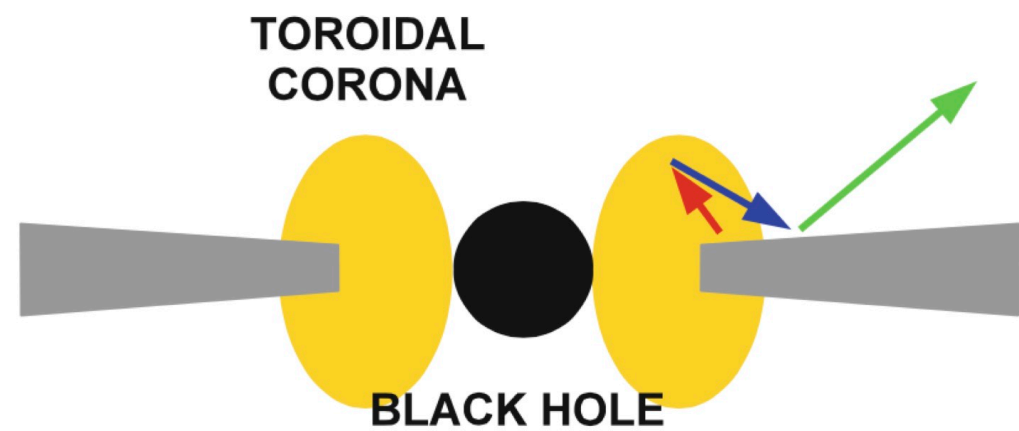
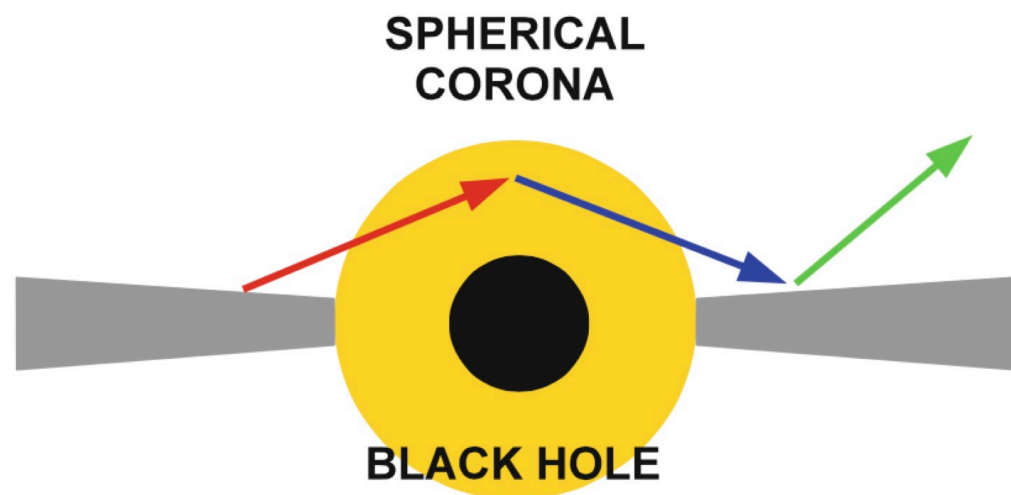
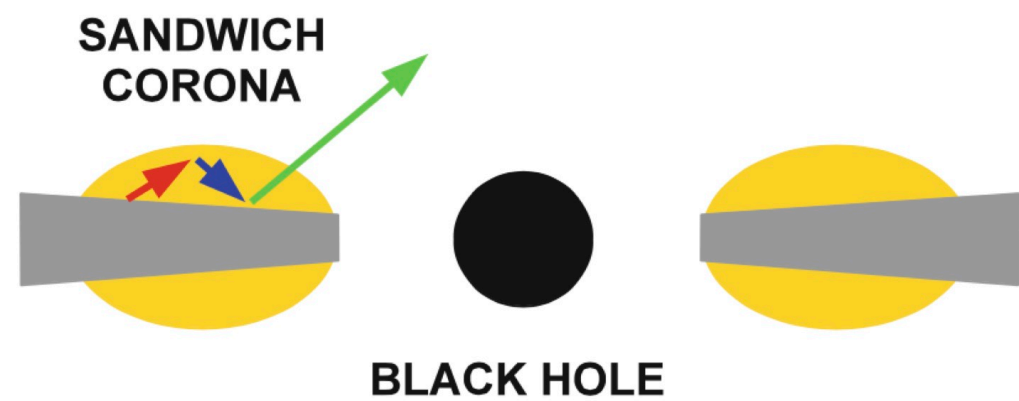
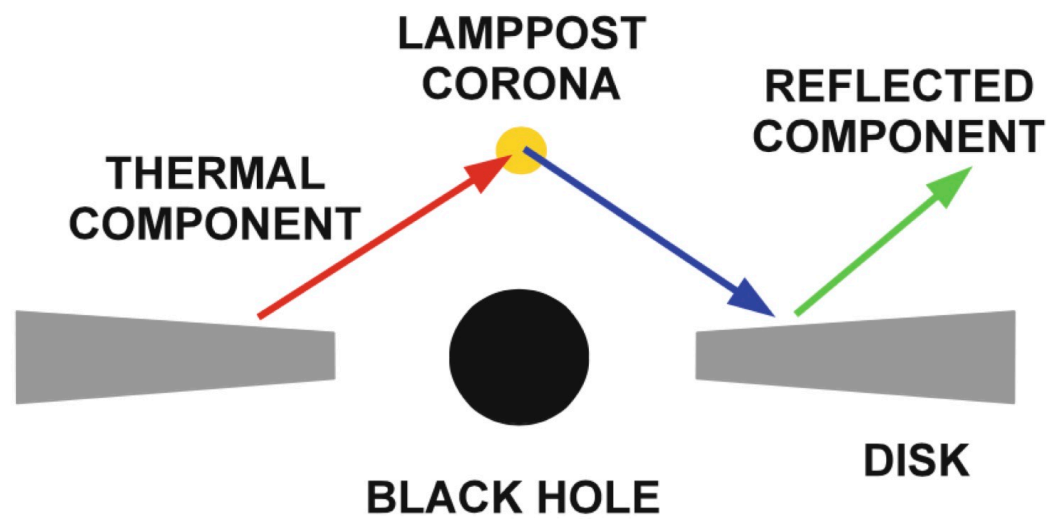
# Disk-Corona Model



# Disk-Corona Model



# Coronal Geometries

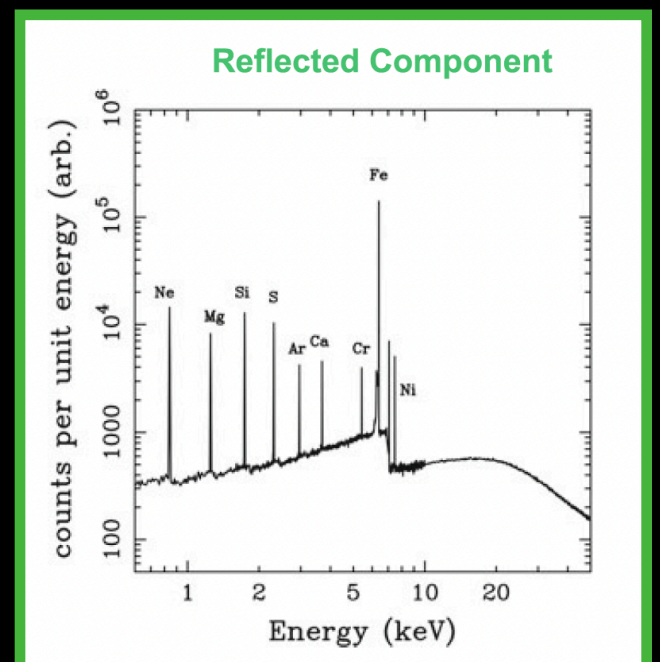
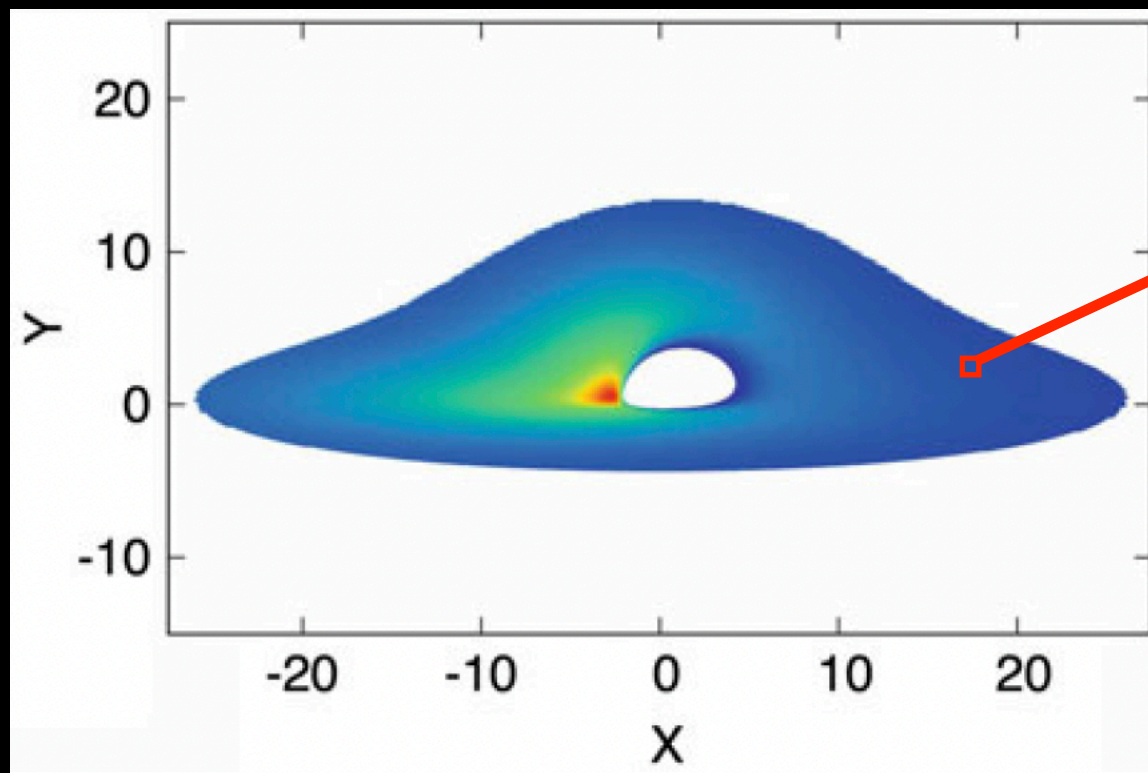




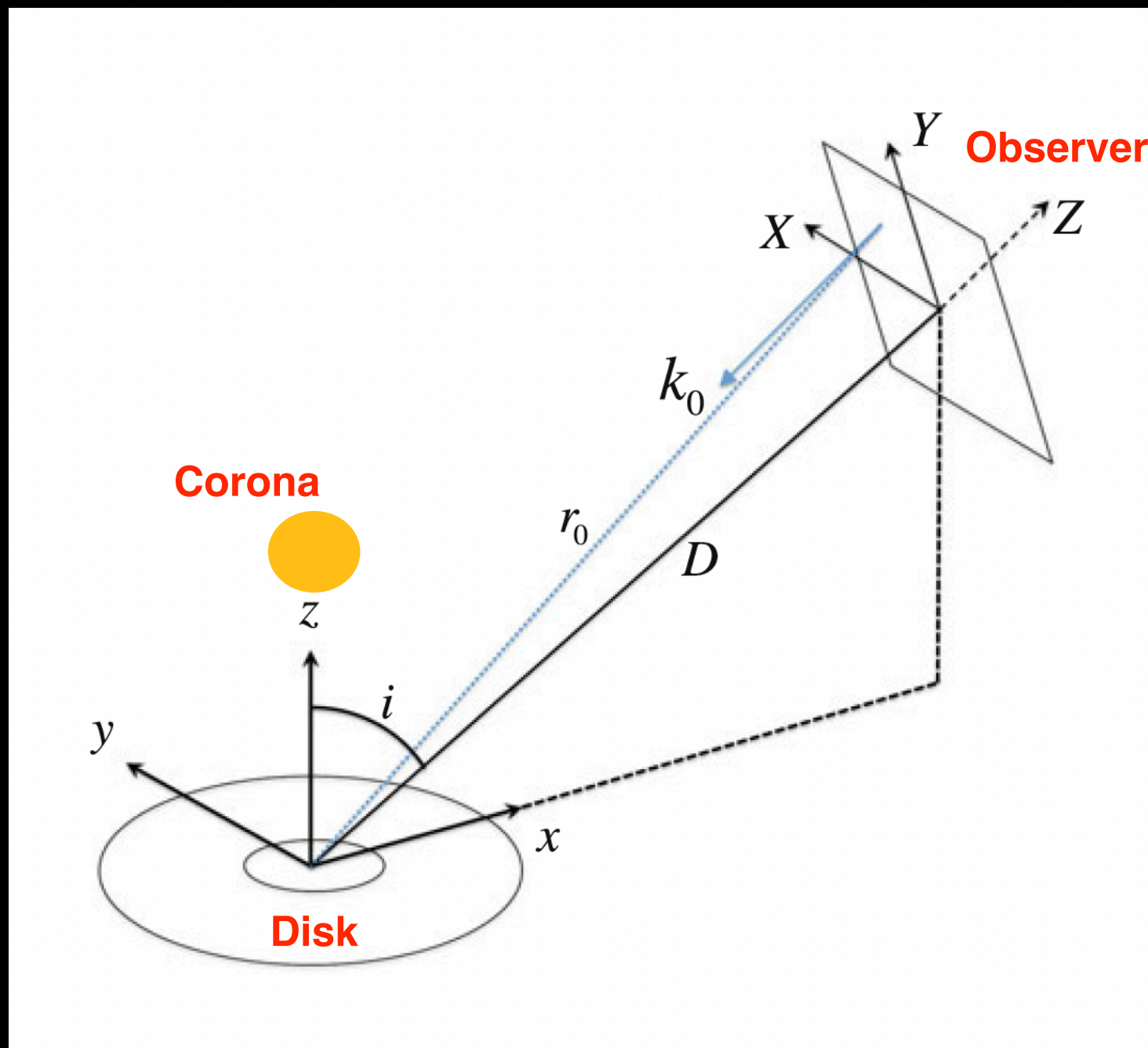
# Synthetic Spectra

# Observed Flux

$$F_{\text{obs}}(\nu_{\text{obs}}) = \int I_{\text{obs}}(\nu_{\text{obs}}, X, Y) d\tilde{\Omega} = \int g^3 I_e(\nu_e, r_e, \vartheta_e) d\tilde{\Omega} ,$$



# Disk-Observer System



# Redshift Image of the Disk

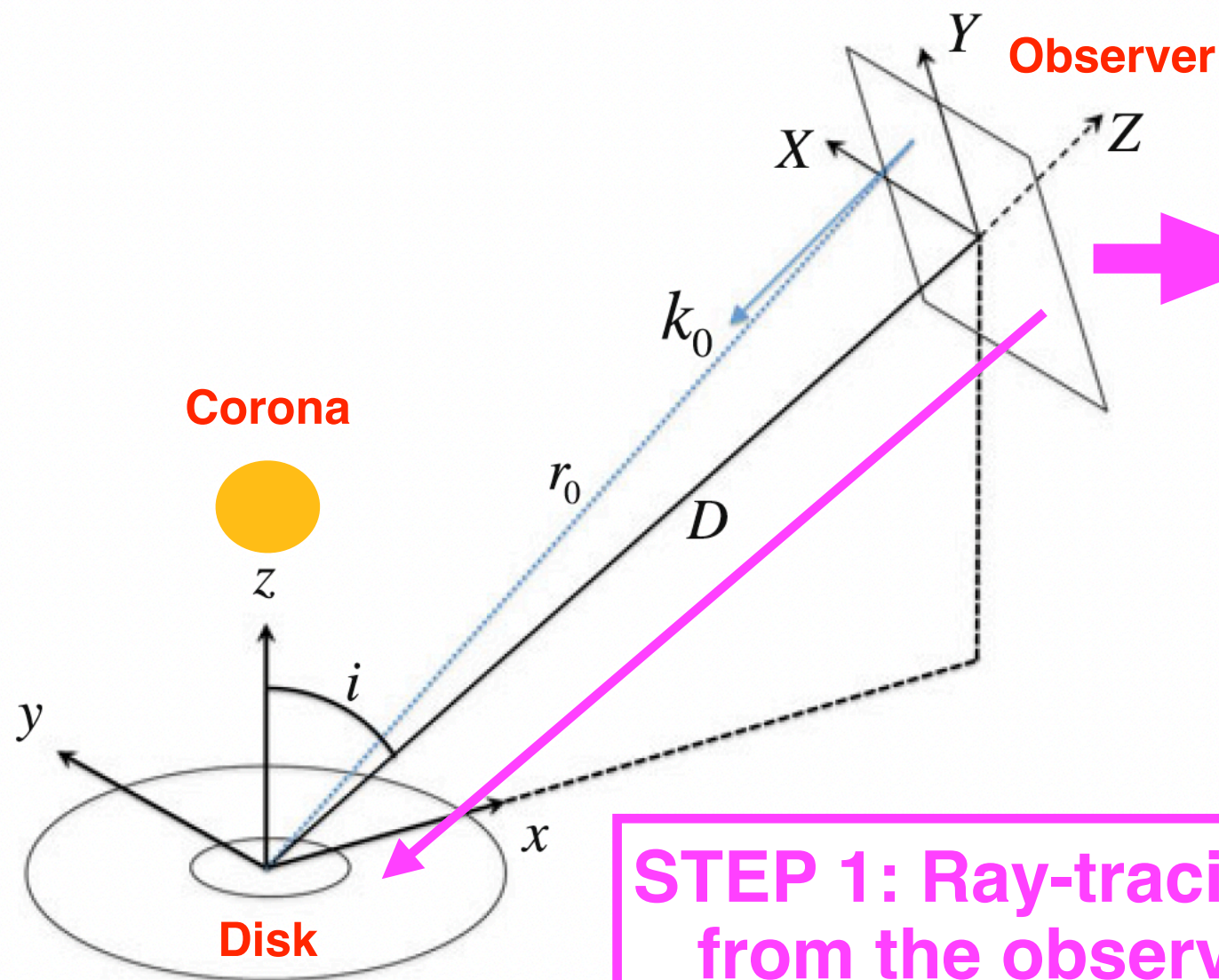
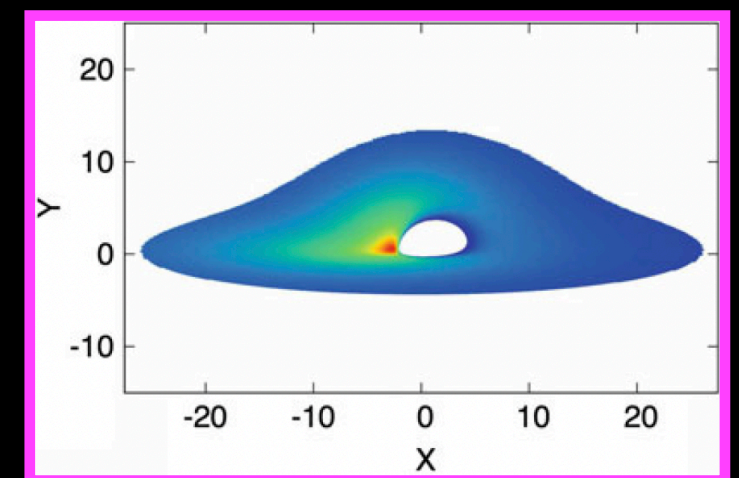


Image of the accretion disk



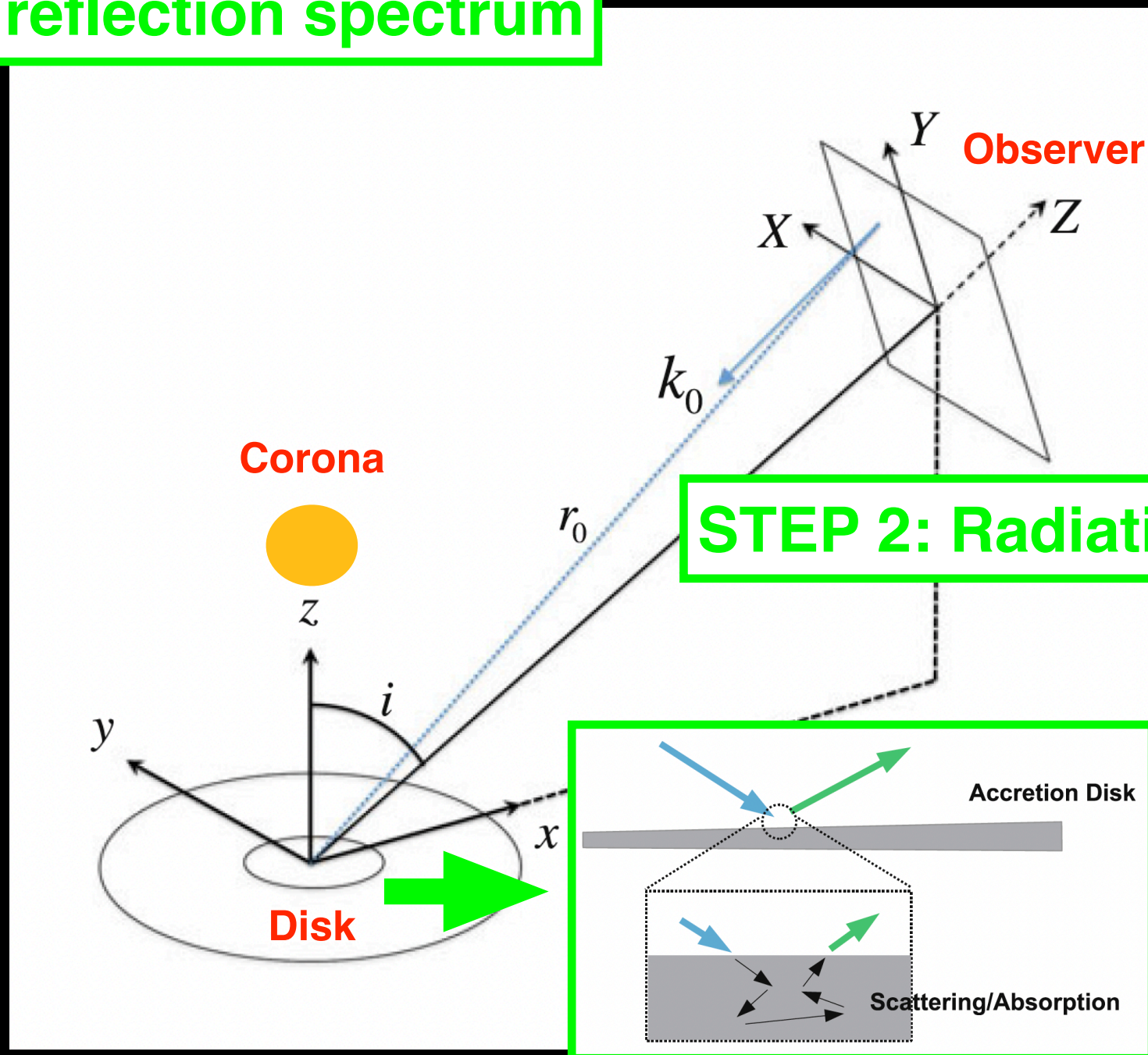
Every point on the image is characterized by its redshift factor  $g$

**STEP 1: Ray-tracing calculations from the observer to the disk**

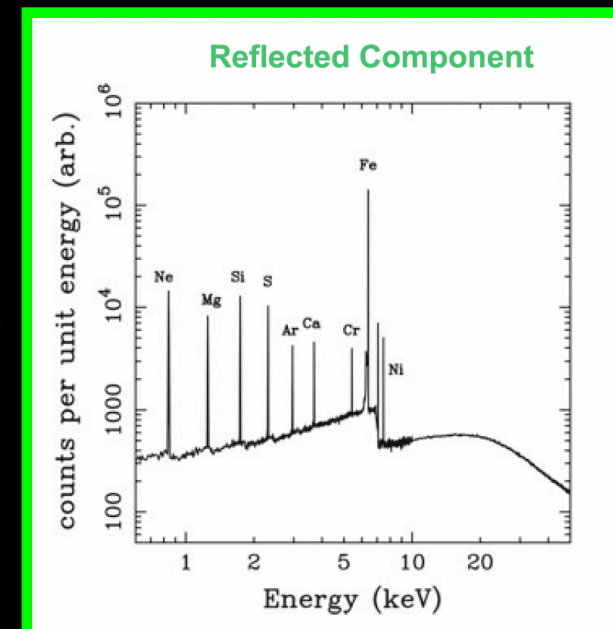


# Local Spectrum

For reflection spectrum

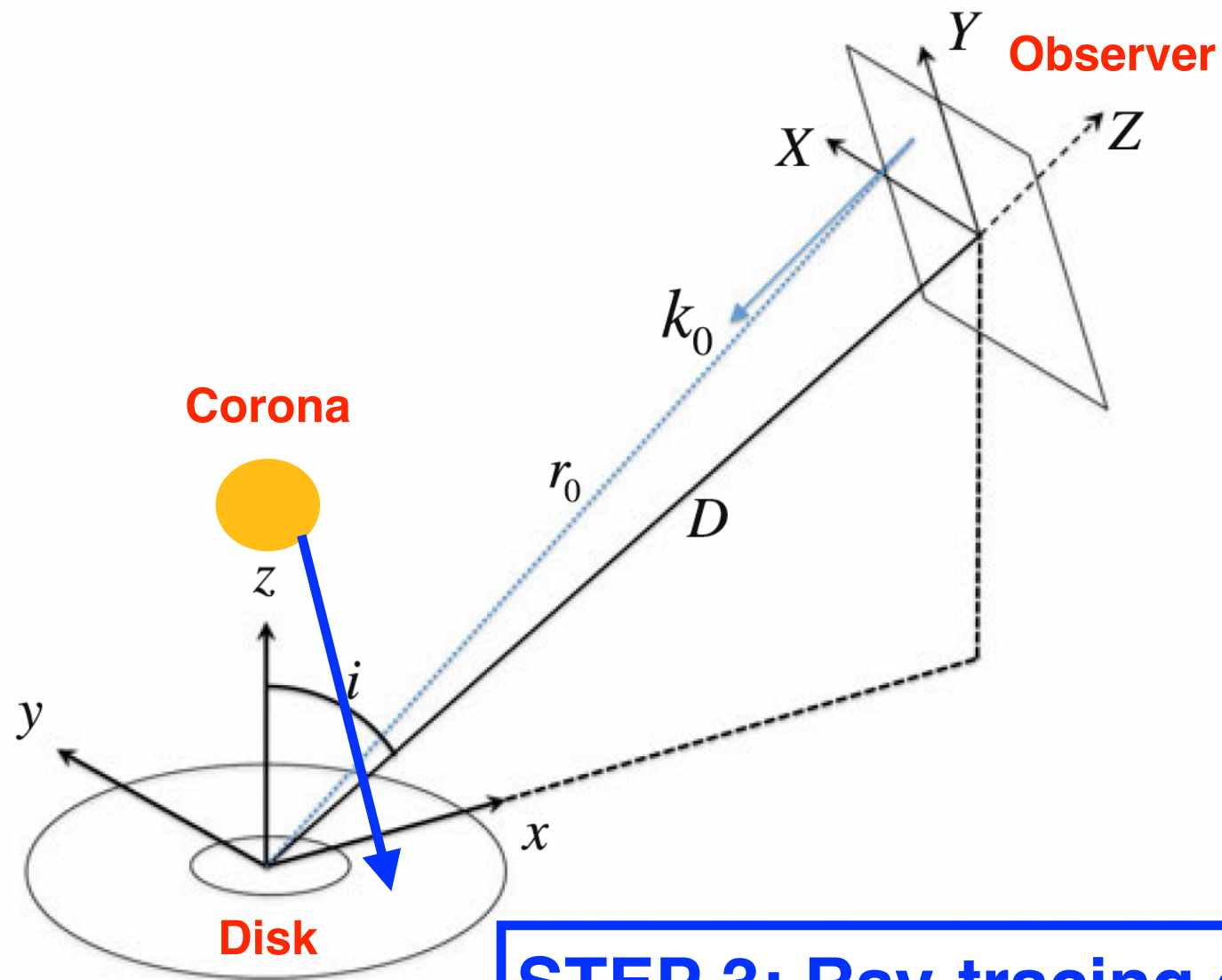


STEP 2: Radiative transfer equations



# Emissivity Profile of the Disk

For reflection spectrum



**STEP 3: Ray-tracing calculations  
from the corona to the disk**




# Transfer Function

$$F_{\text{obs}}(\nu_{\text{obs}}) = \int I_{\text{obs}}(\nu_{\text{obs}}, X, Y) d\tilde{\Omega} = \int g^3 I_{\text{e}}(\nu_{\text{e}}, r_{\text{e}}, \vartheta_{\text{e}}) d\tilde{\Omega} ,$$



$$F_{\text{obs}}(\nu_{\text{obs}}) = \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \pi r_{\text{e}} \frac{g^2}{\sqrt{g^*(1-g^*)}} f(g^*, r_{\text{e}}, i) I_{\text{e}}(\nu_{\text{e}}, r_{\text{e}}, \vartheta_{\text{e}}) dg^* dr_{\text{e}} .$$

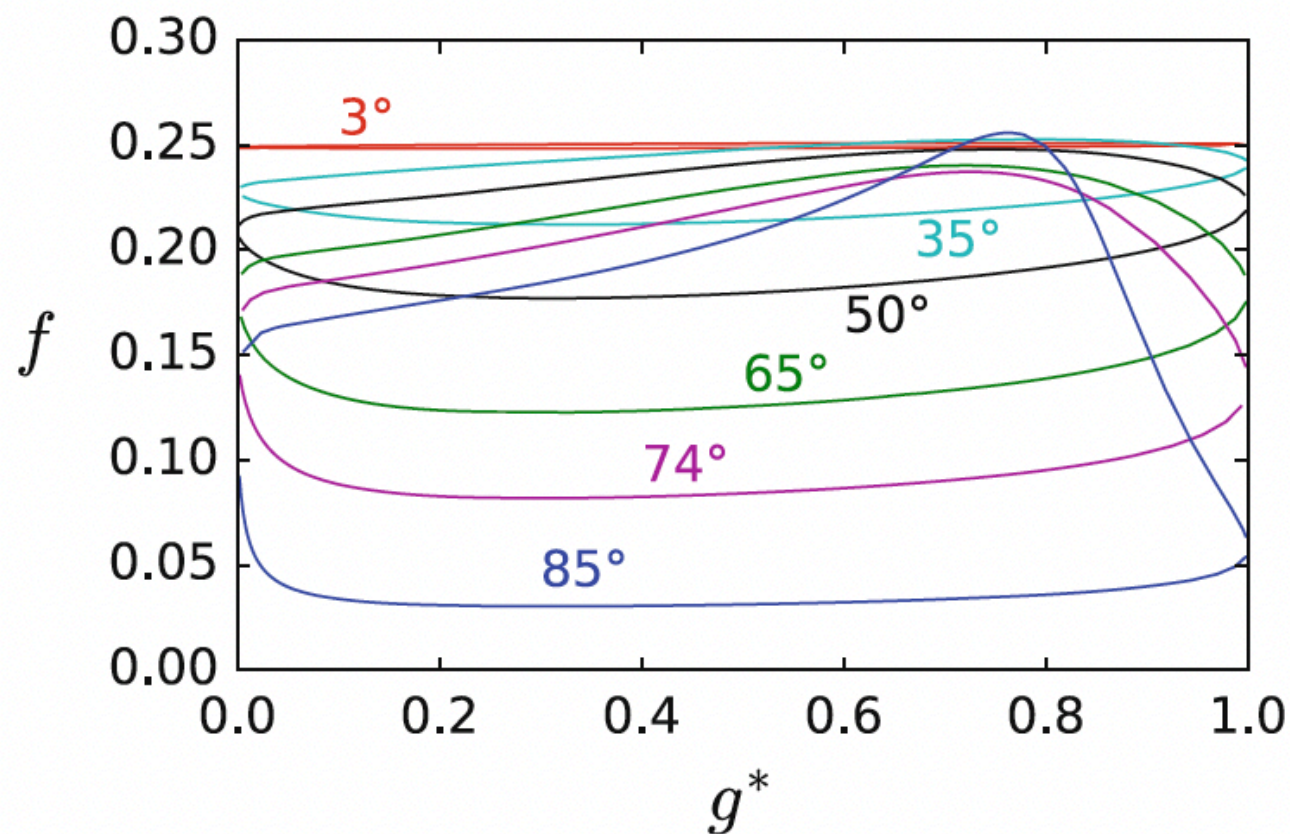

$$g^* = \frac{g - g_{\text{min}}}{g_{\text{max}} - g_{\text{min}}} ,$$



$$f(g^*, r_{\text{e}}, i) = \frac{1}{\pi r_{\text{e}}} g \sqrt{g^*(1-g^*)} \left| \frac{\partial (X, Y)}{\partial (g^*, r_{\text{e}})} \right| ,$$

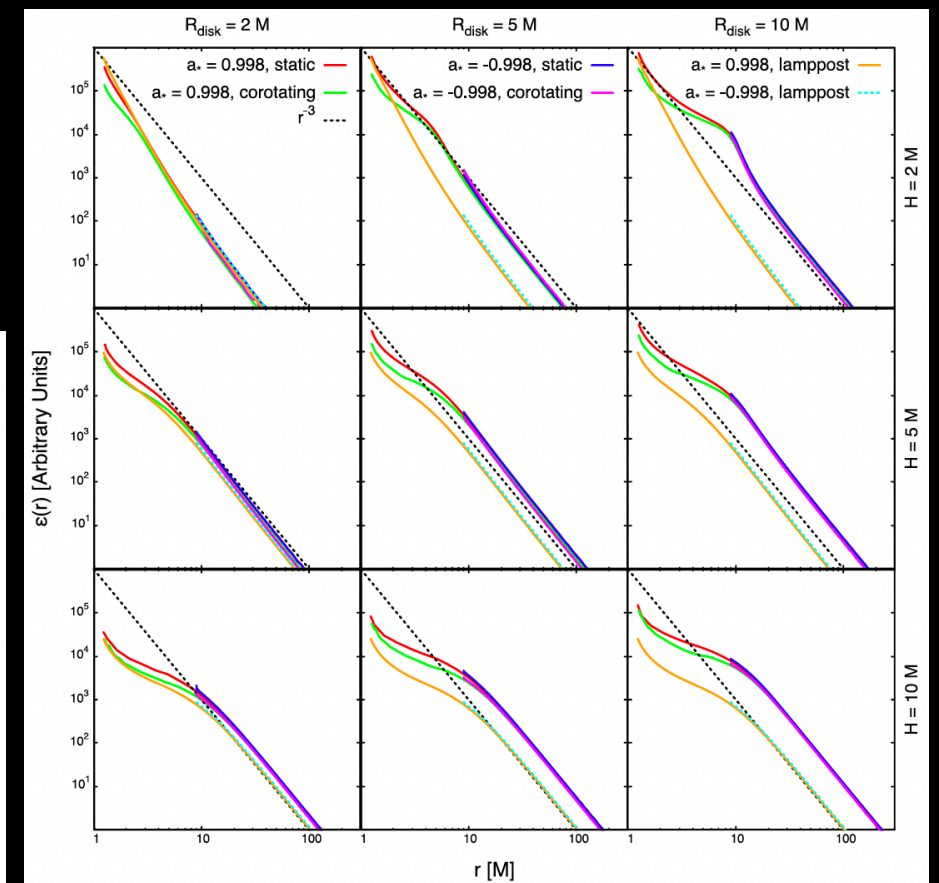
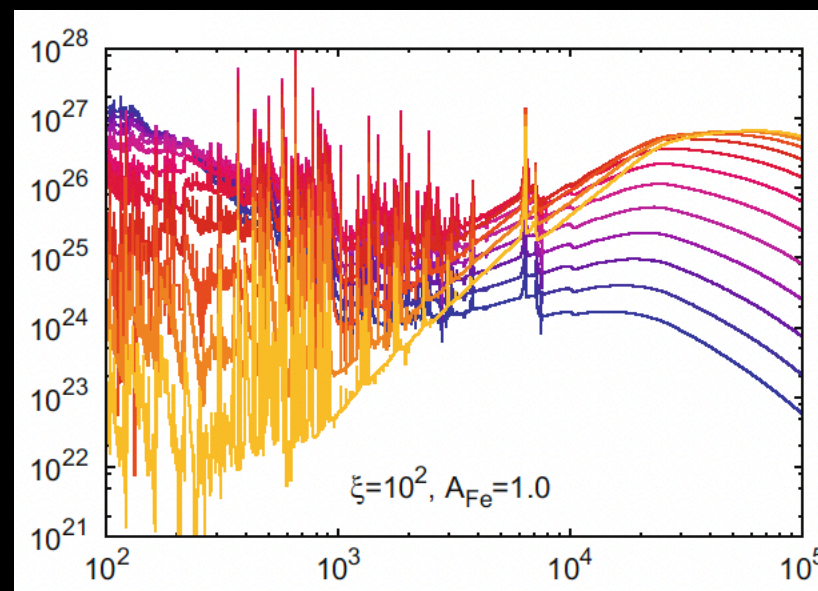
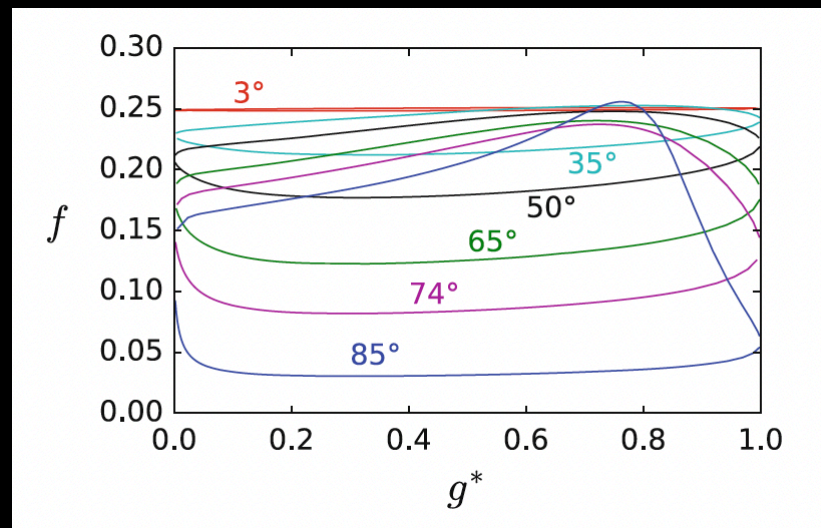
# Transfer Function

$$F_{\text{obs}}(\nu_{\text{obs}}) = \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_1(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,1}) dg^* dr_e \\ + \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_2(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,2}) dg^* dr_e ,$$



# FITS Files

$$F_{\text{obs}}(\nu_{\text{obs}}) = \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_1(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,1}) dg^* dr_e \\ + \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_2(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,2}) dg^* dr_e ,$$

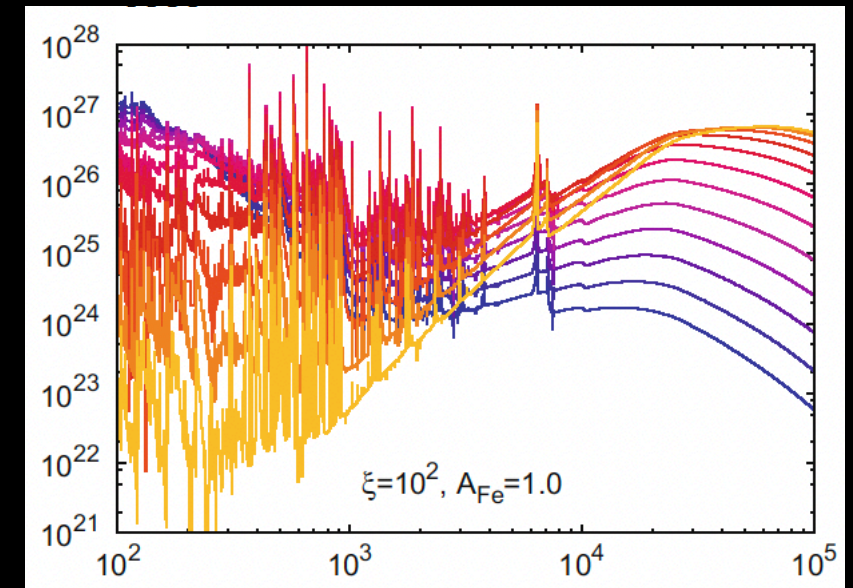


# Reflection Models



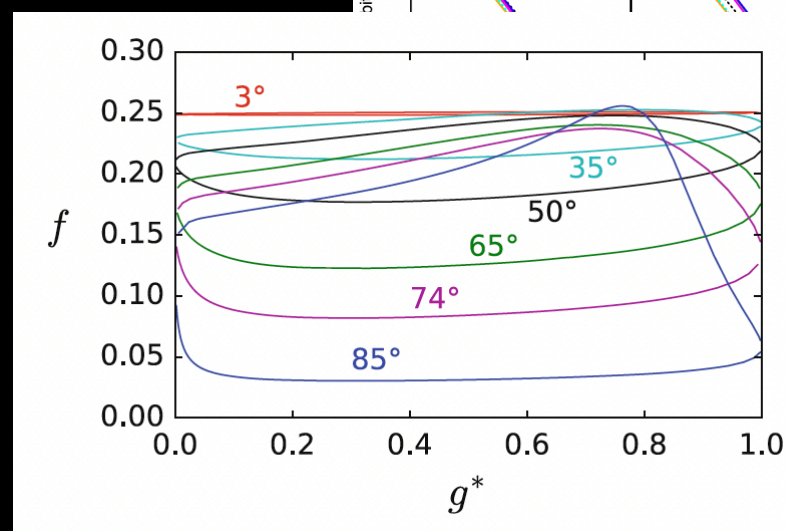
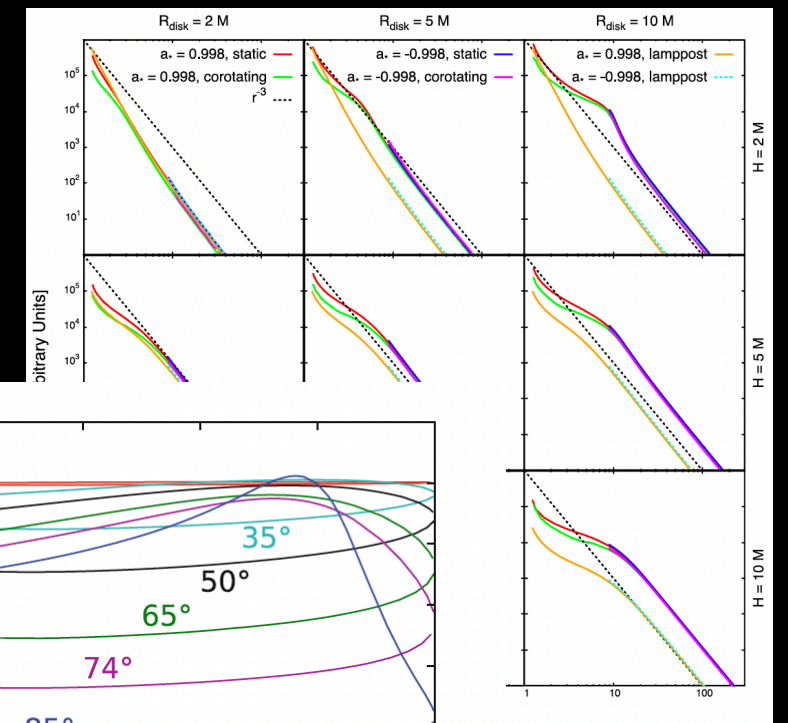
# Reflection Models

- Non-relativistic reflection models:
  - `relionx` (Ross & Fabian)
  - `xillver` (Garcia & Kallman)
  - `ireflect` (Magdziarz & Zdziarski)



- Relativistic reflection models:

- `relxill` (Dauser & Garcia)
  - `relxill_nk` (Fudan)
- `kyn` (Dovciak)
- `reflkerr` (Niedzwiecki)
- `reltrans` (Ingram)



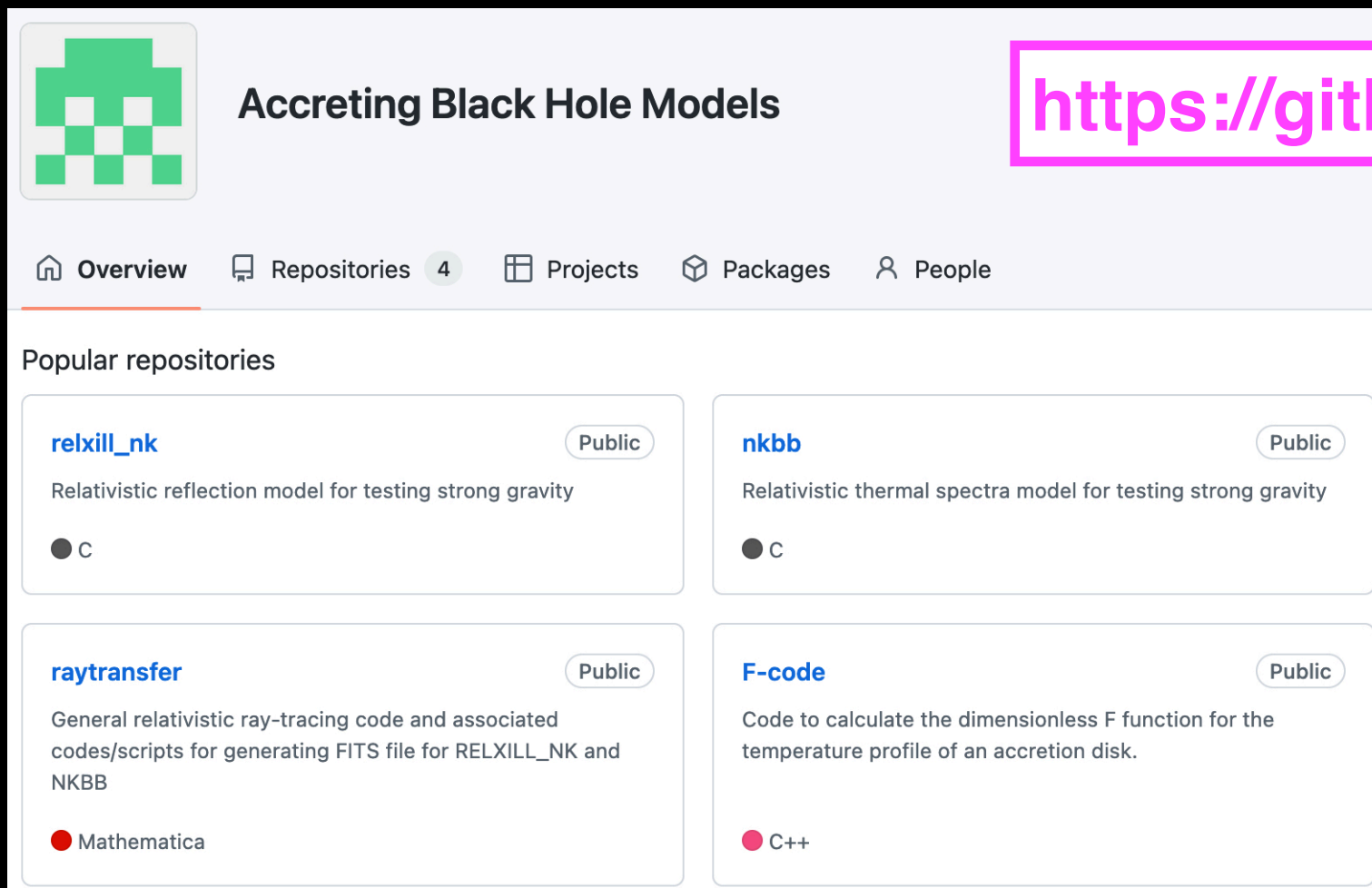
# ABHModels

- **relxill\_nk** (Bambi et al. 2017; Abdikamalov et al. 2019)

Reflection spectrum for thin accretion disks in stationary, axisymmetric, and asymptotically-flat spacetimes

- **nkbb** (Zhou et al. 2019)

Thermal spectrum for thin accretion disks in stationary, axisymmetric, and asymptotically-flat spacetimes



The screenshot shows the GitHub repository page for 'Accreting Black Hole Models'. The repository is public and contains four sub-repositories: **relxill\_nk** (C), **nkbb** (C), **raytransfer** (Mathematica), and **F-code** (C++). The page includes a navigation bar with 'Overview', 'Repositories' (4), 'Projects', 'Packages', and 'People'. The 'Popular repositories' section lists the four sub-repositories with their descriptions and programming languages.

Repository Name	Language	Description
<b>relxill_nk</b>	C	Relativistic reflection model for testing strong gravity
<b>nkbb</b>	C	Relativistic thermal spectra model for testing strong gravity
<b>raytransfer</b>	Mathematica	General relativistic ray-tracing code and associated codes/scripts for generating FITS file for RELXILL_NK and NKBB
<b>F-code</b>	C++	Code to calculate the dimensionless F function for the temperature profile of an accretion disk.

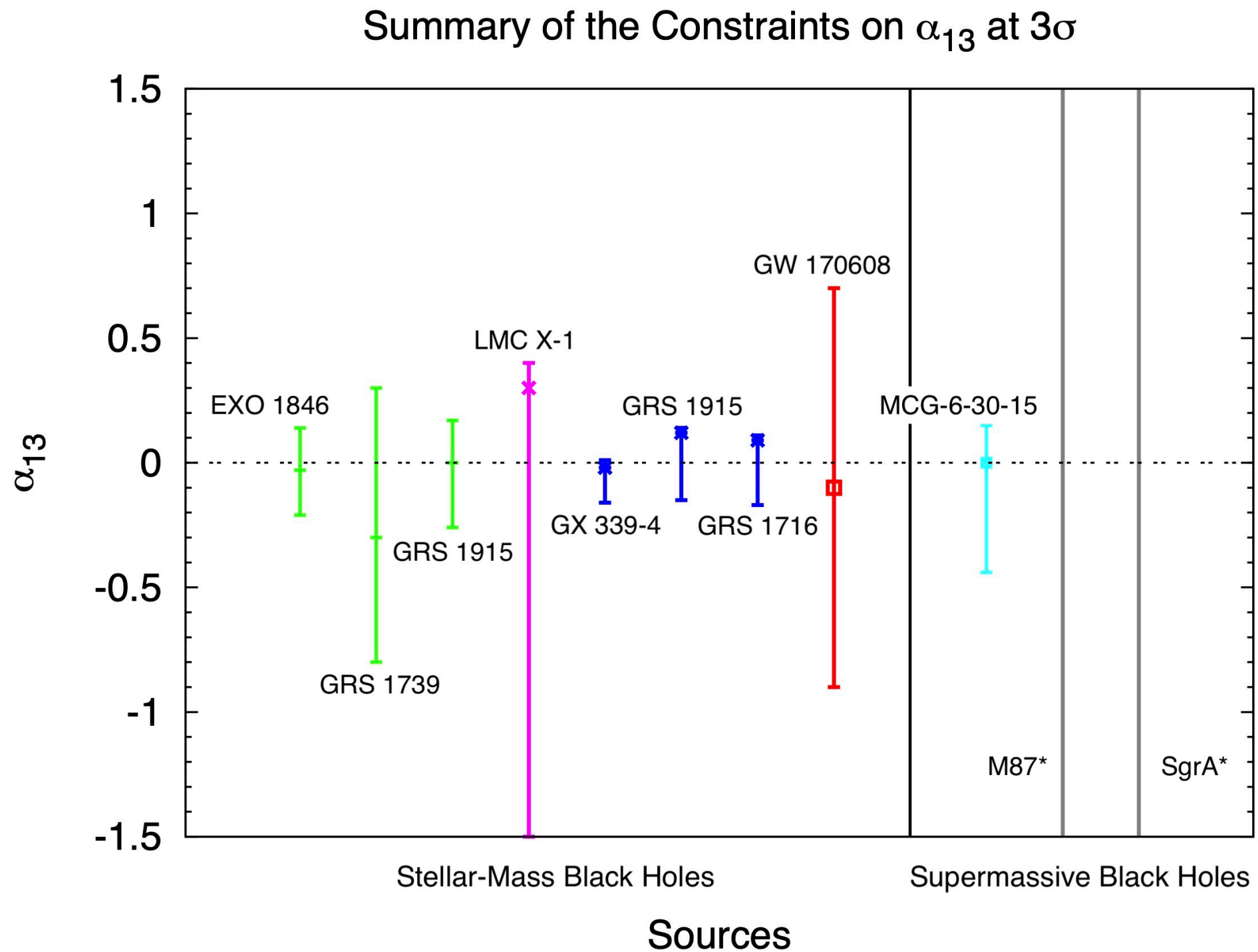
<https://github.com/ABHModels>



# GR Tests with XRS

- **Tests of the Kerr black hole hypothesis**
  - Agnostic tests
  - Tests of specific gravity models (we need to know the rotating black hole solution of the theory that we want to test)
- Tests of geodesic motion/tests of the Weak Equivalence Principle
  - Non-minimal couplings between the matter and gravity sectors
- Tests of variation of fundamental constants
  - The atomic physics in strong gravitational fields is different from the atomic physics in our laboratories on Earth

# Tests of the Kerr Hypothesis



## **Part II:**

- 1) Are current XRS measurements of black holes accurate?**
- 2) How can we improve current reflection models?**

# Are Current XRS Measurements of Black Holes Accurate?

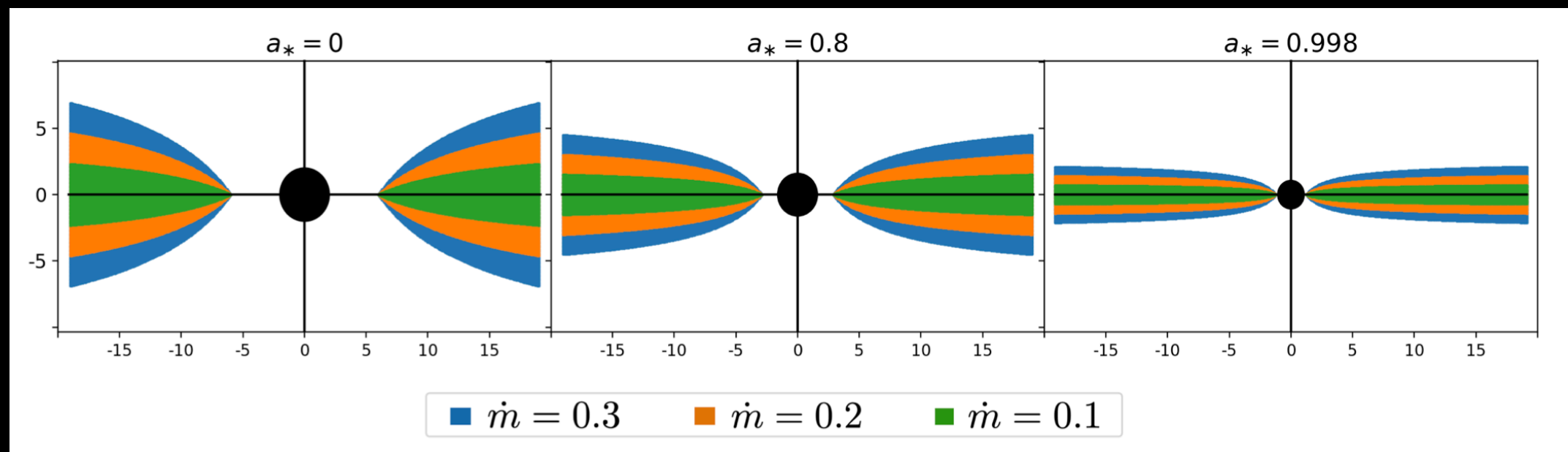
Quick (and personal) answer: yes, but only if we select the right sources and observations

- High spin ( $a > 0.9$ )
- Compact corona close to the black hole
- Prominent and broadened iron line
- $L \sim 0.05-0.30 L_{\text{Edd}}$
- High resolution at the iron line + hard X-ray band

# Disk Structure

# Thickness of the Disk

- Thin disks of finite thickness (from Taylor & Reynolds 2018)
- Abdikamalov et al. 2020 (model, GRS 1915+105)
- Tripathi et al. 2021 (MCG-6-30-15, EXO 1846-031)
- Jiang et al. 2022 (lampost corona, MCG-6-30-15)

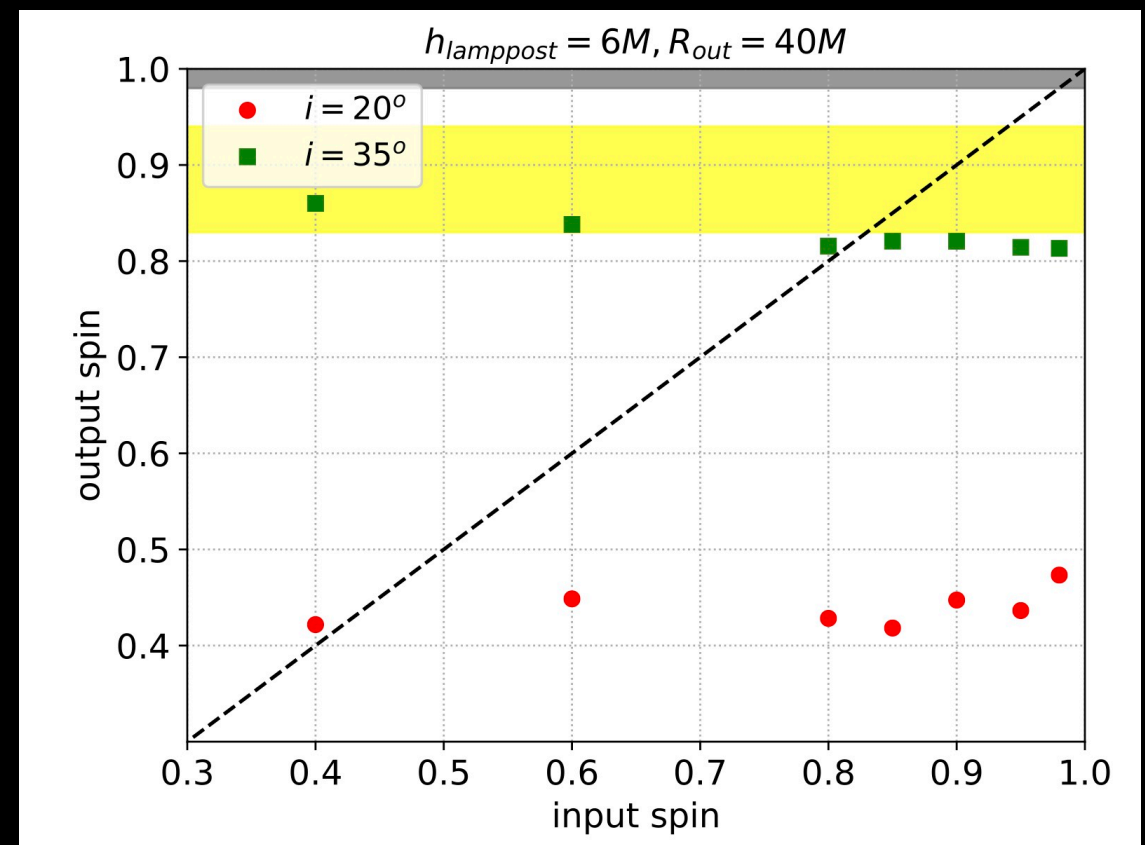
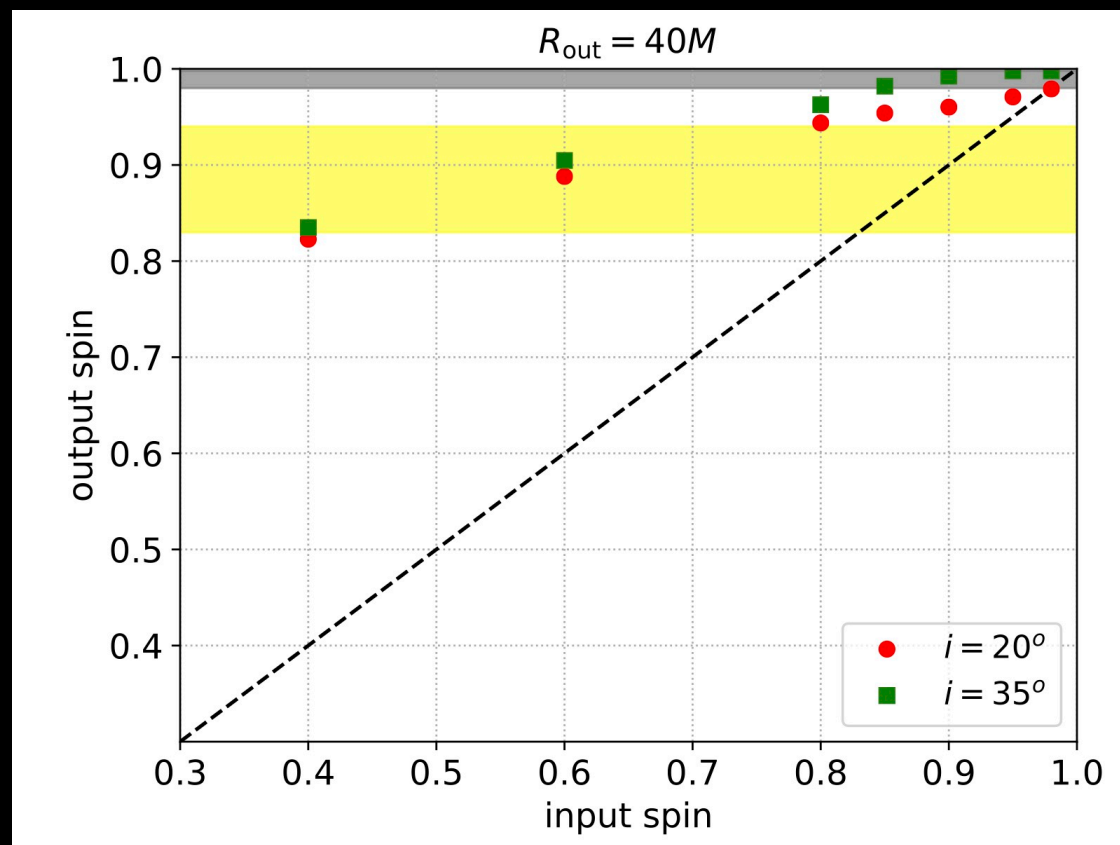


**Conclusion: no significant impact on the parameter estimate for disks with high radiative efficiency**



# Thickness of the Disk

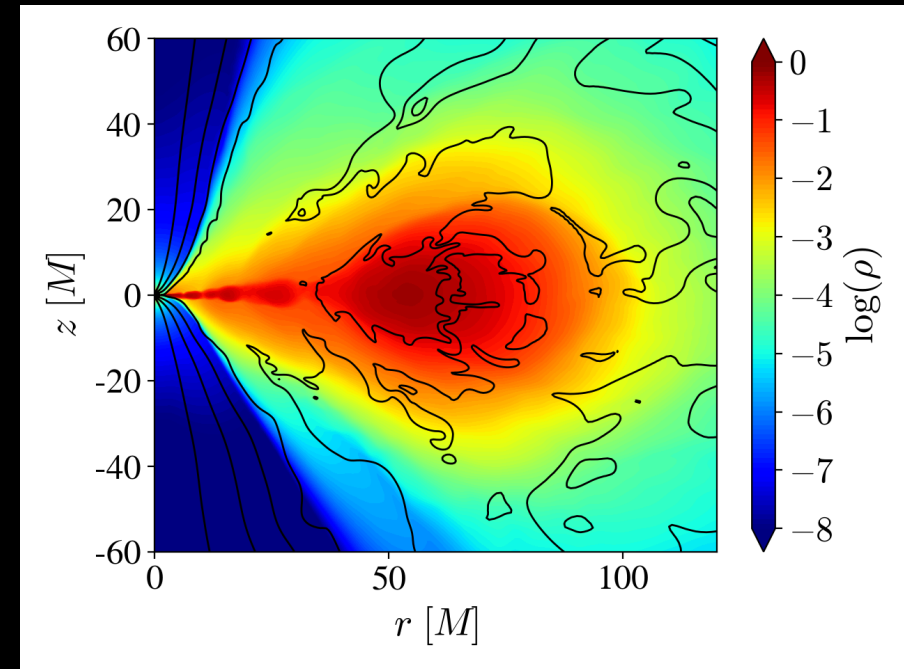
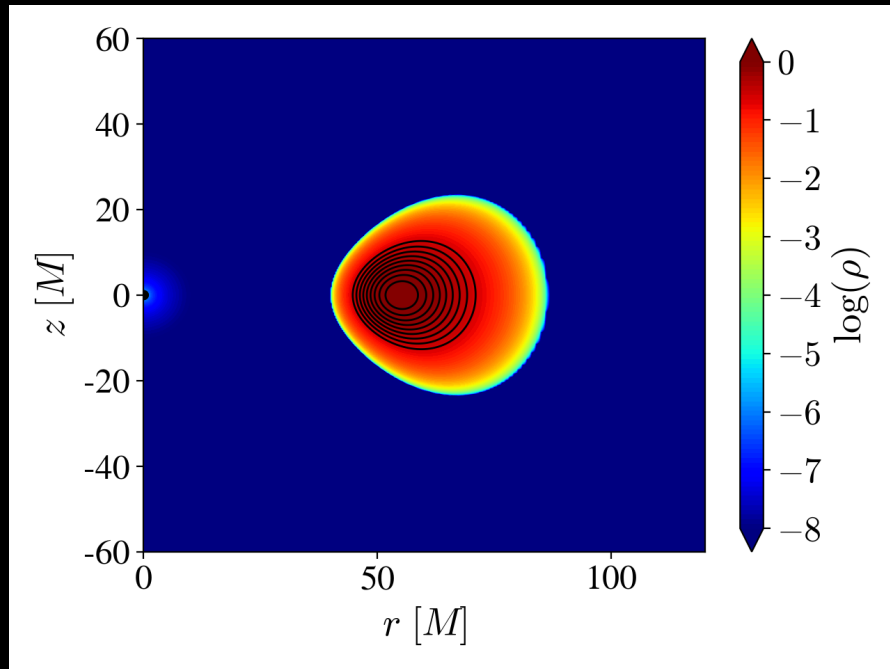
- Thick disks
- Riaz et al. 2020a, 2020b



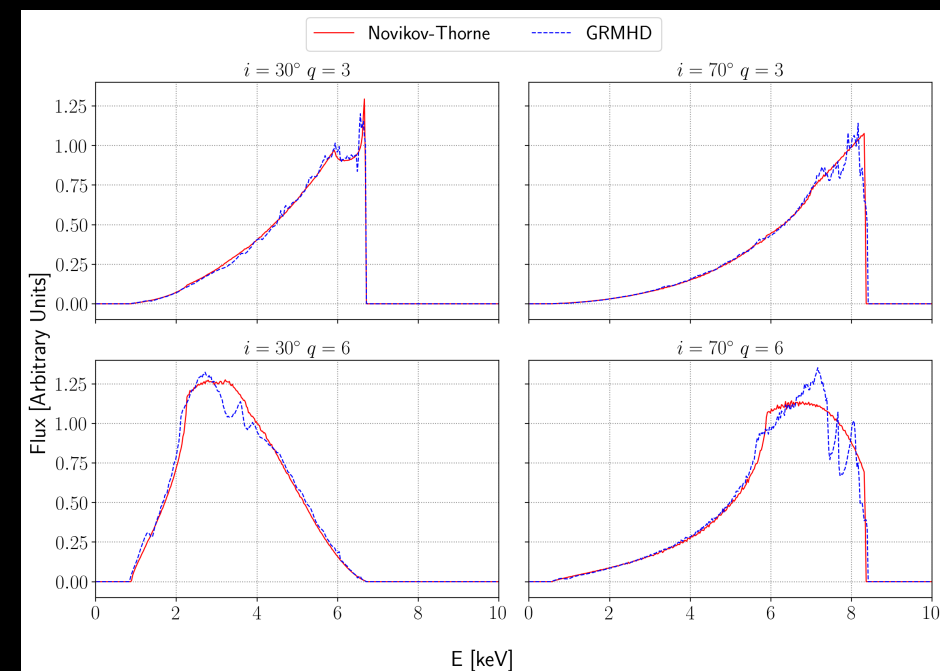
**Conclusion: large systematic uncertainties,  
wrong measurements**

# GRMHD-Simulated Thin Disks

- Shashank et al. 2022

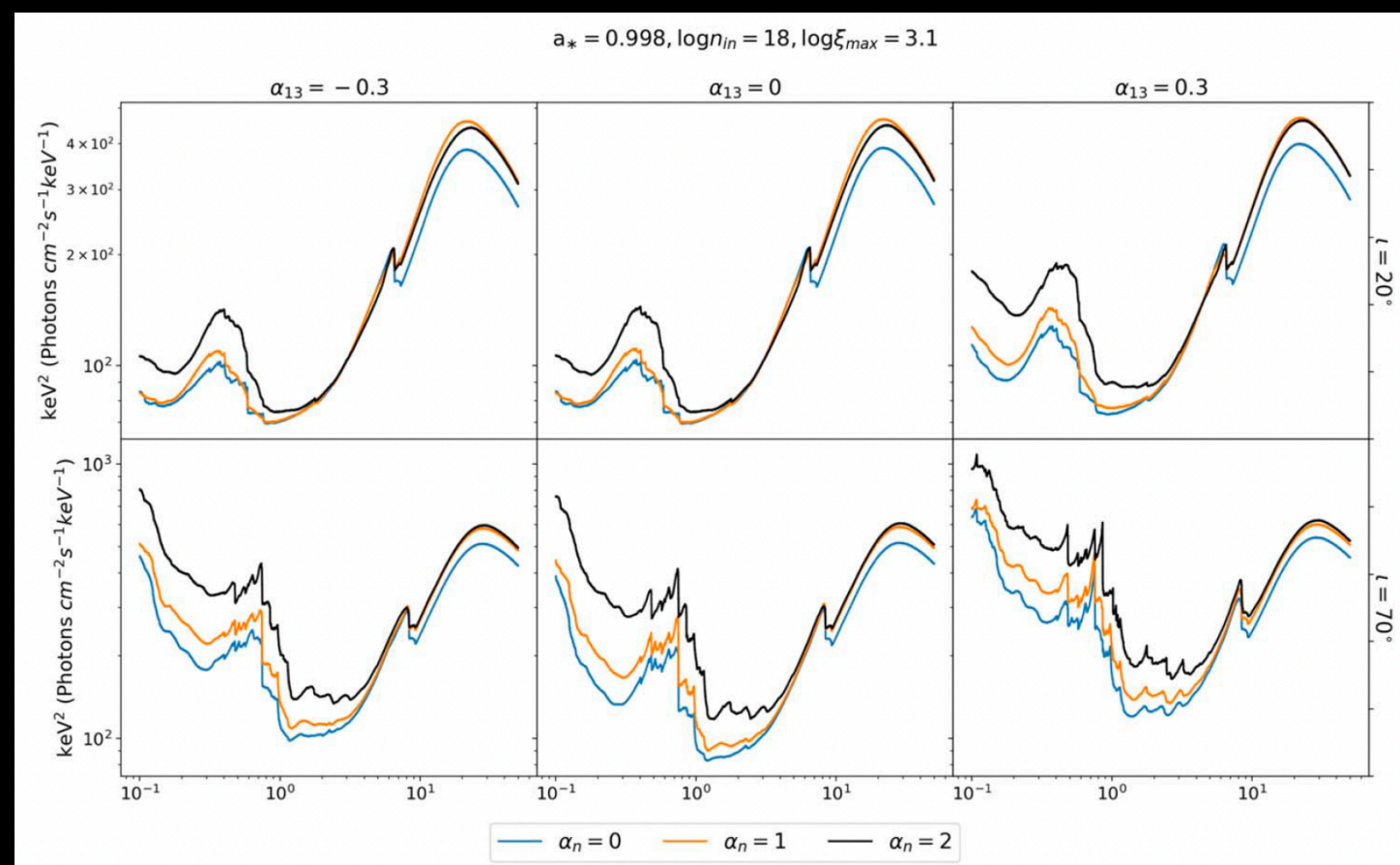


**Conclusion: we recover well the input parameters**



# Ionization and Density Gradients

- Abdikamalov et al. 2021a (relxillion\_nk, EXO 1846-031)
- Abdikamalov et al. 2021b (relxilldgrad\_nk, EXO 1846-031)
- Mall et al. 2022 (GS 1354-645, GRS 1739-278, GRS 1915+105)

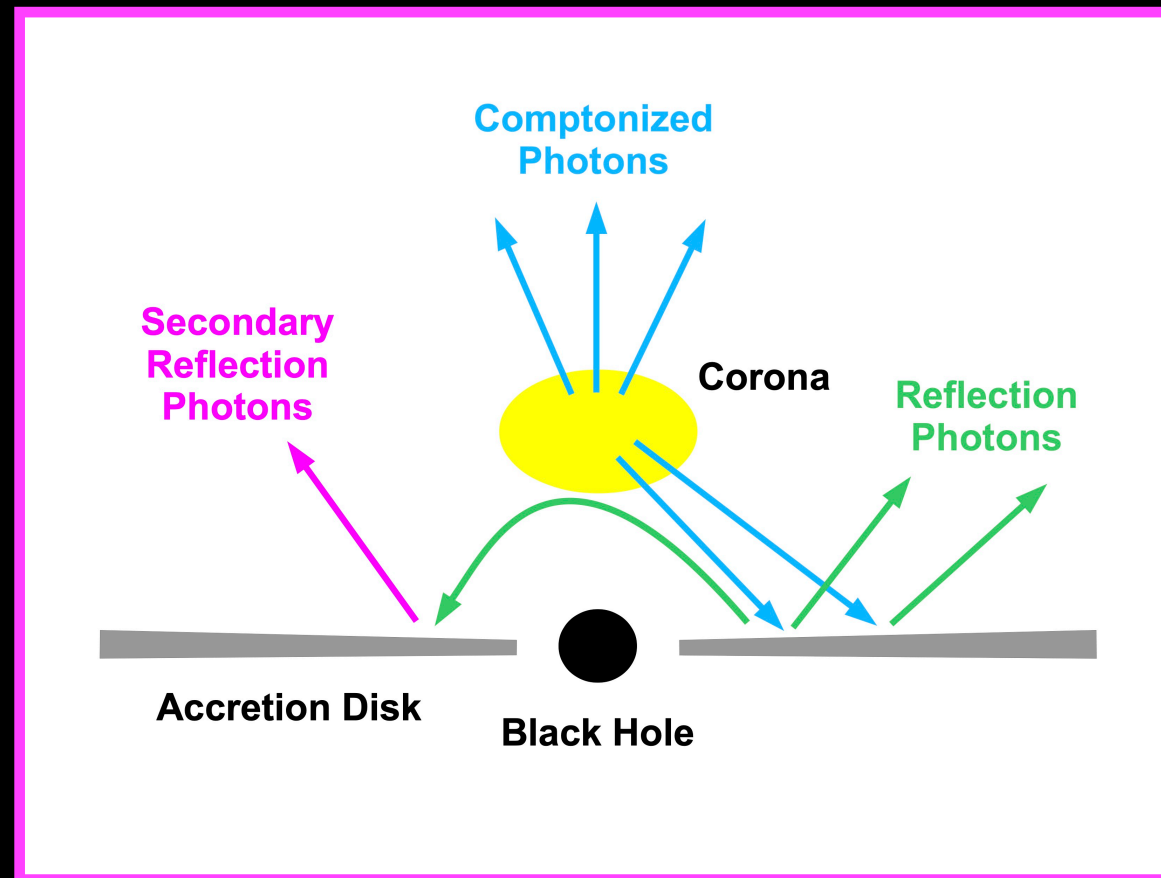


**Conclusion: somewhat better fits,  
but no significant impact on the parameter estimate**

# Returning Radiation

# Returning Radiation

- Mirzaev et al. in preparation



**Conclusion: we can have large systematic uncertainties when:**

- 1) The black hole spin is high**
- 2) The corona is compact and close to the black hole**
- 3) The ionization parameter is not high**

**Plunging Region**



# Plunging Region

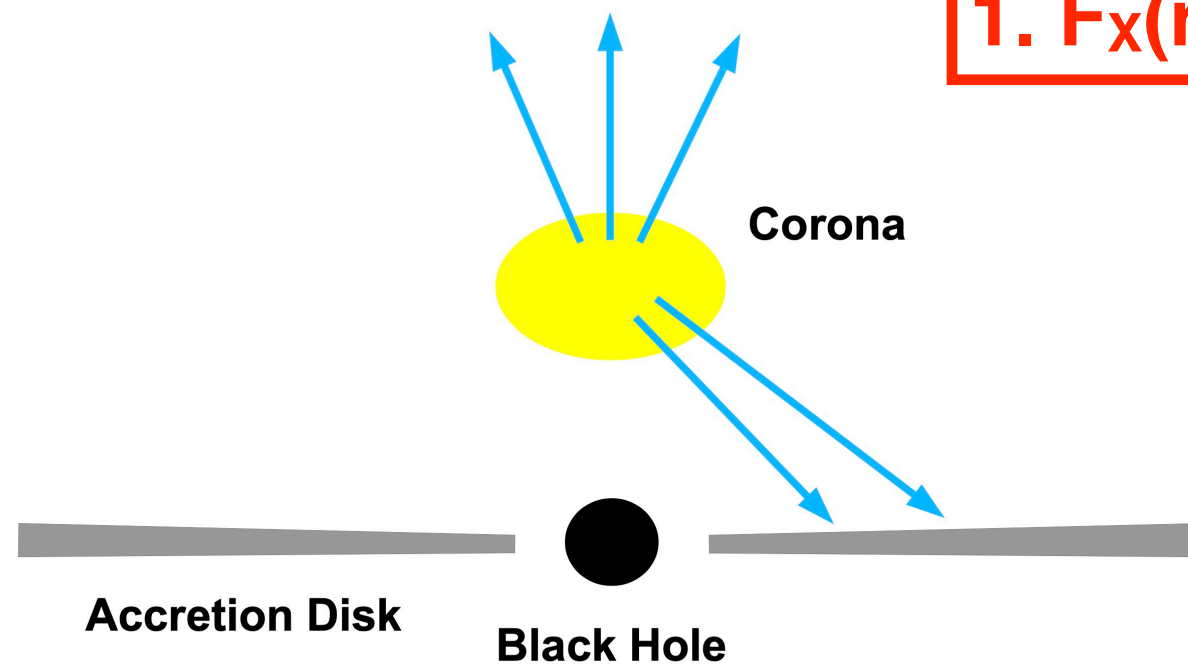
- Plunging region is optically thin:
  - Higher order disk images (Zhou et al. 2020)
- Plunging region is optically thick:
  - Reflection spectrum from the plunging region (Cardenas-Avendano et al. 202)

**Conclusion: no significant impact on the parameter estimate,  
especially for fast-rotating black holes**

# **How Can We Improve Current Reflection Models?**

# shuaa

- 1) Spacetime (Kerr:  $M, a$ )
- 2) Disk ( $n$ )
- 3) Corona (lamppost:  $h, \Gamma, T$ )

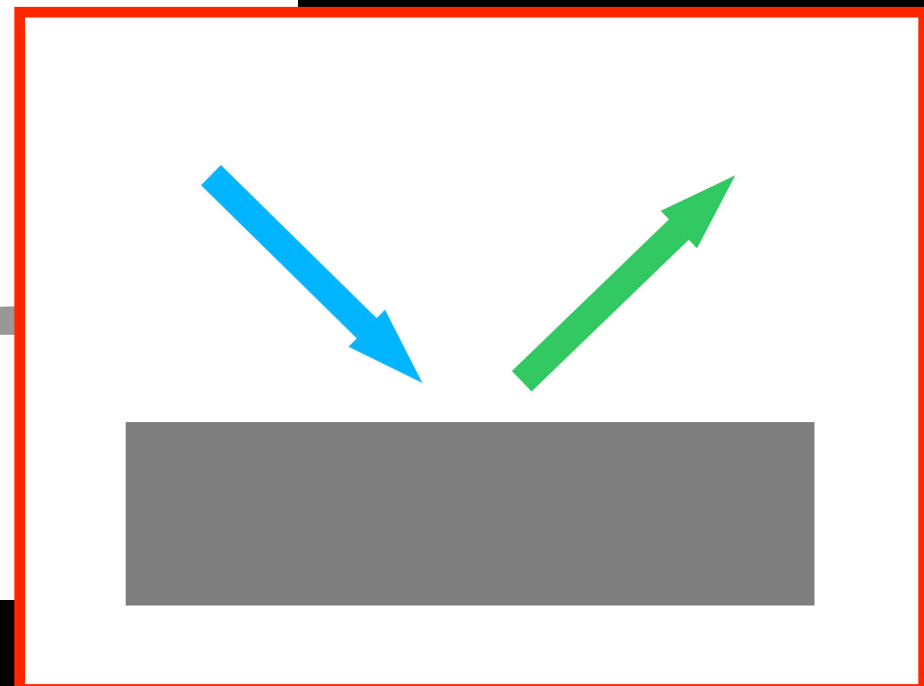
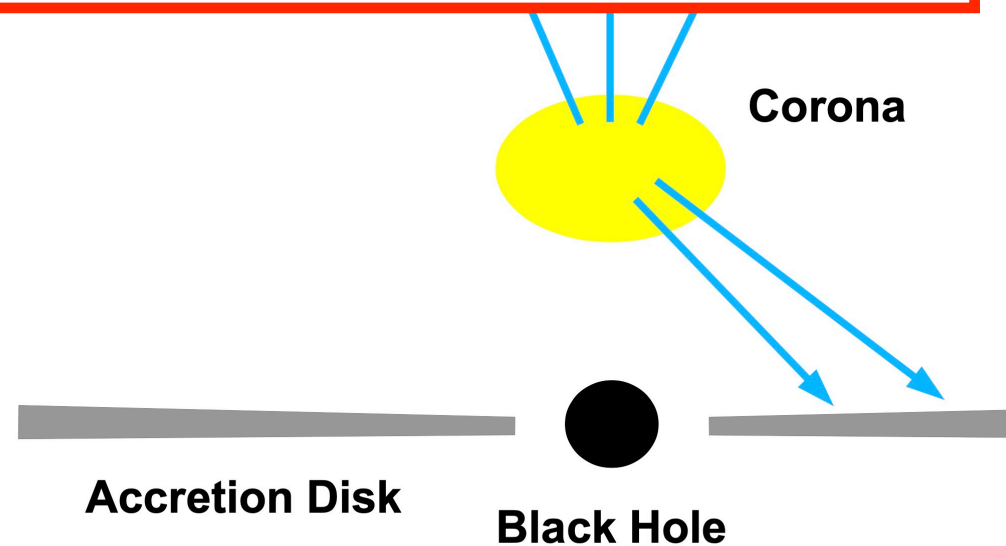


1.  $F_x(r)$  from the corona

# shuaa

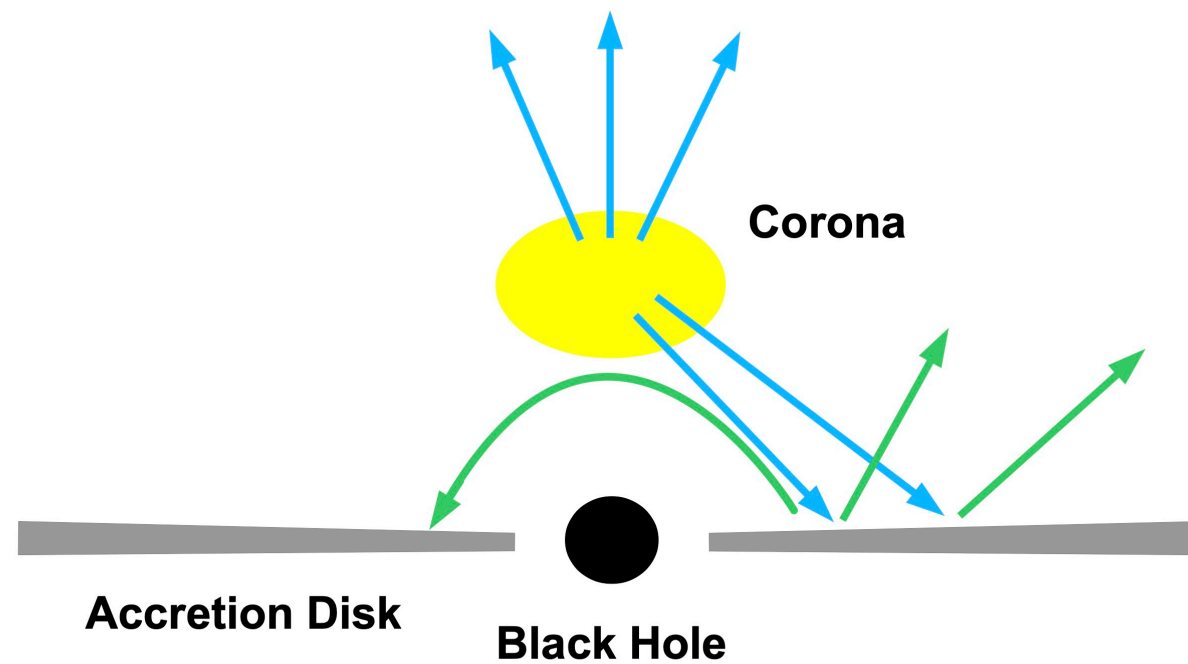
- 1) Spacetime (Kerr:  $M, a$ )
- 2) Disk ( $n$ )
- 3) Corona (lamppost:  $h, \Gamma, T$ )

2. Reflection spectrum  $I_e(r)$   
Note:  $\xi(r)$ , redshifted coronal spectrum



# shuaa

## 3. Returning radiation at every radius

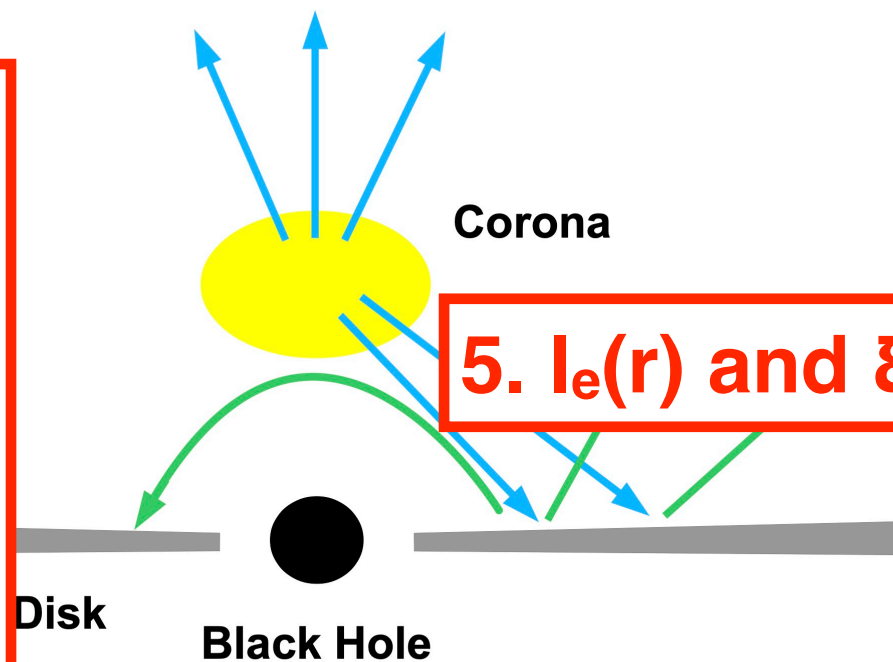
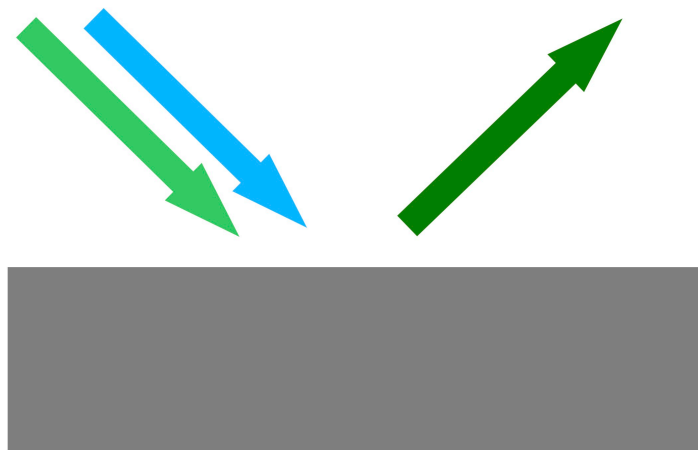


# shuaa

3. Returning radiation at every radius

4. New  $I_e(r)$  and  $\xi(r)$

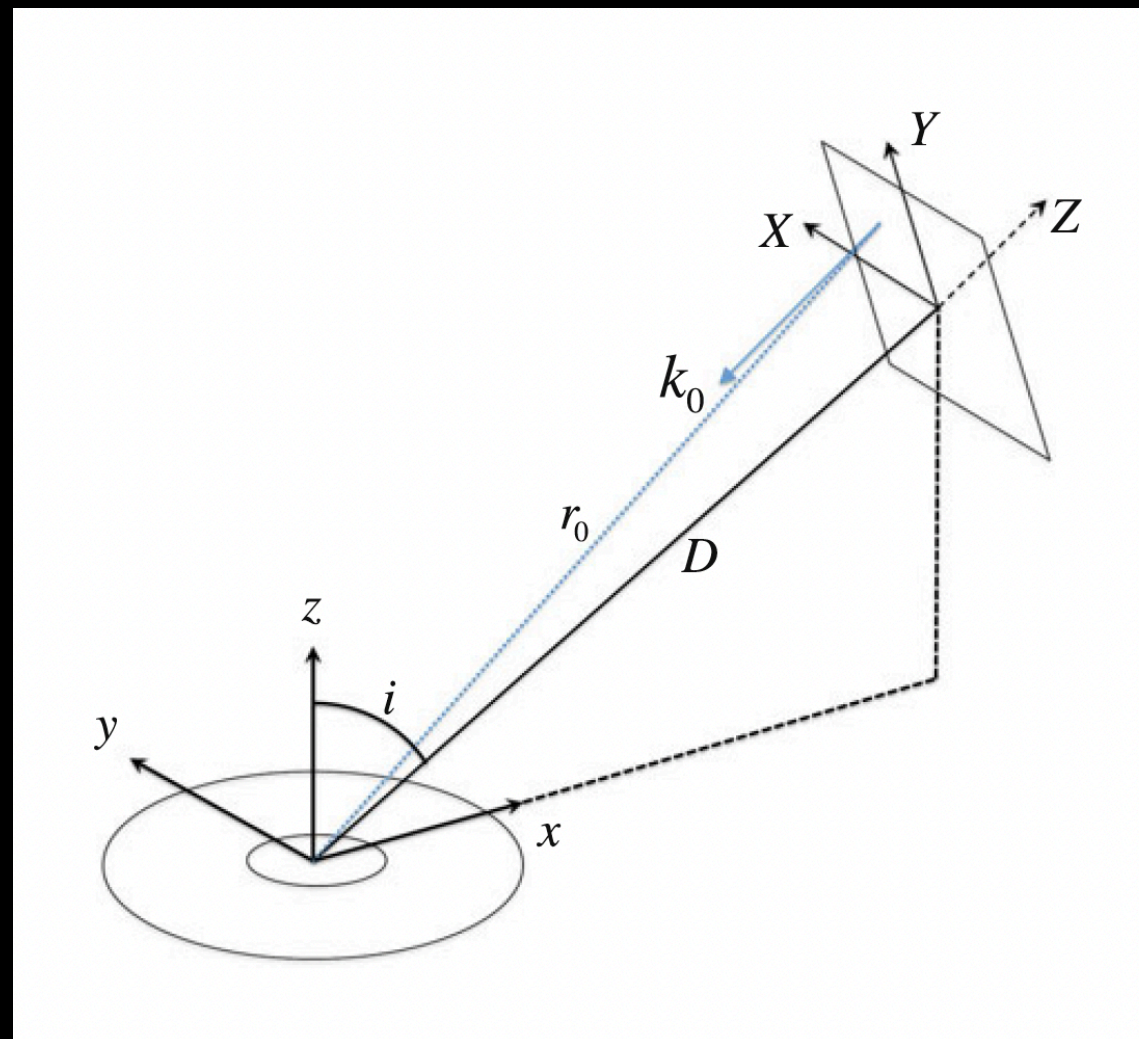
5.  $I_e(r)$  and  $\xi(r)$  at the 4th order





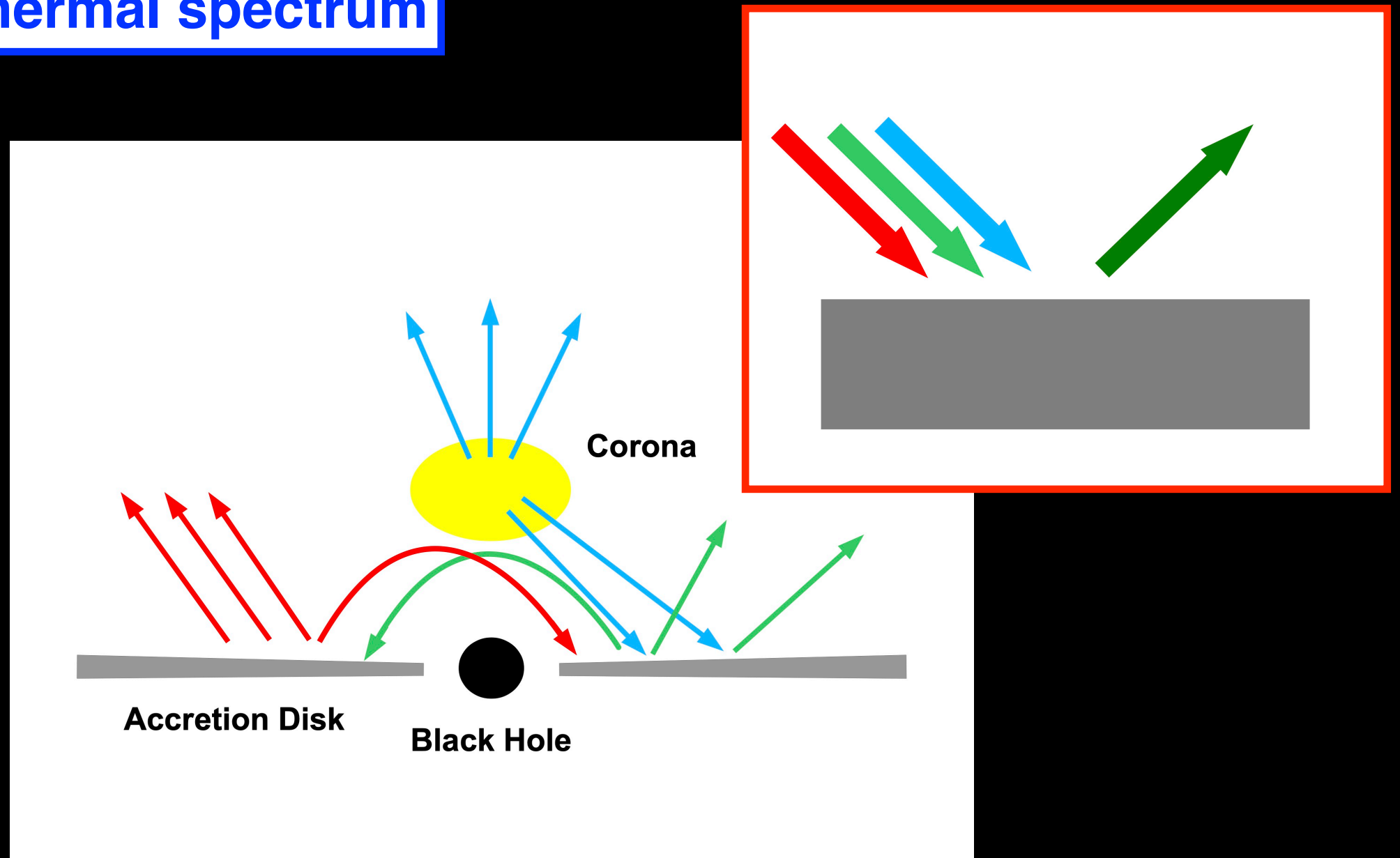
# shuaa

## 6. Observed reflection spectrum (viewing angle $i$ )



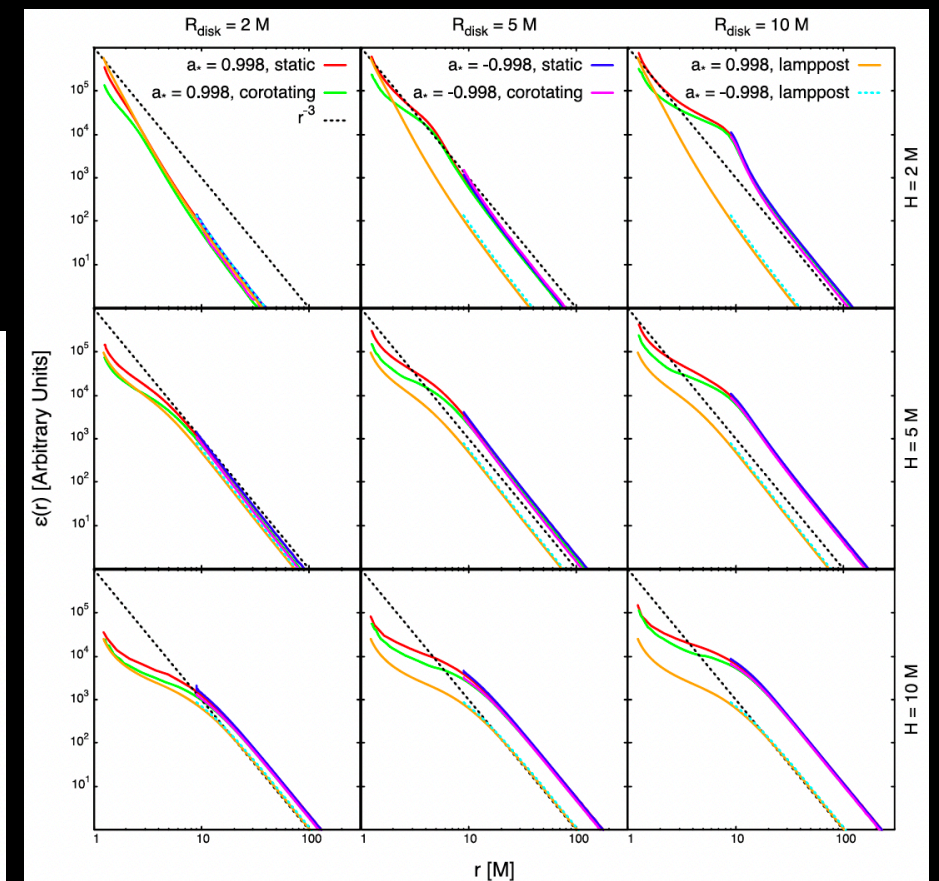
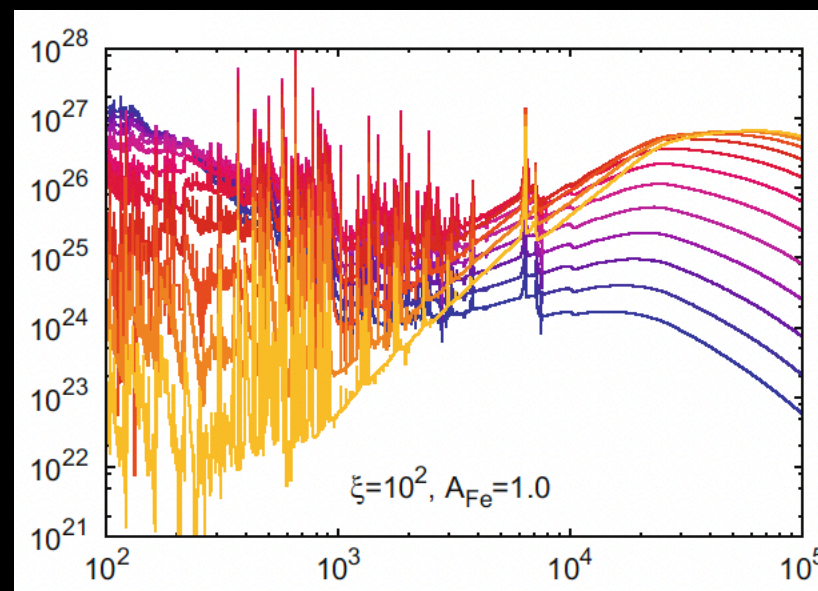
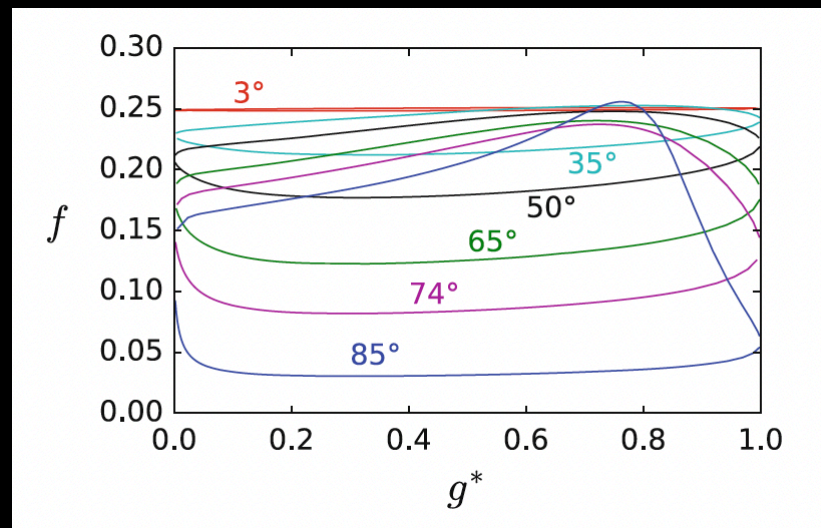
# shuaa

## 7. Thermal spectrum



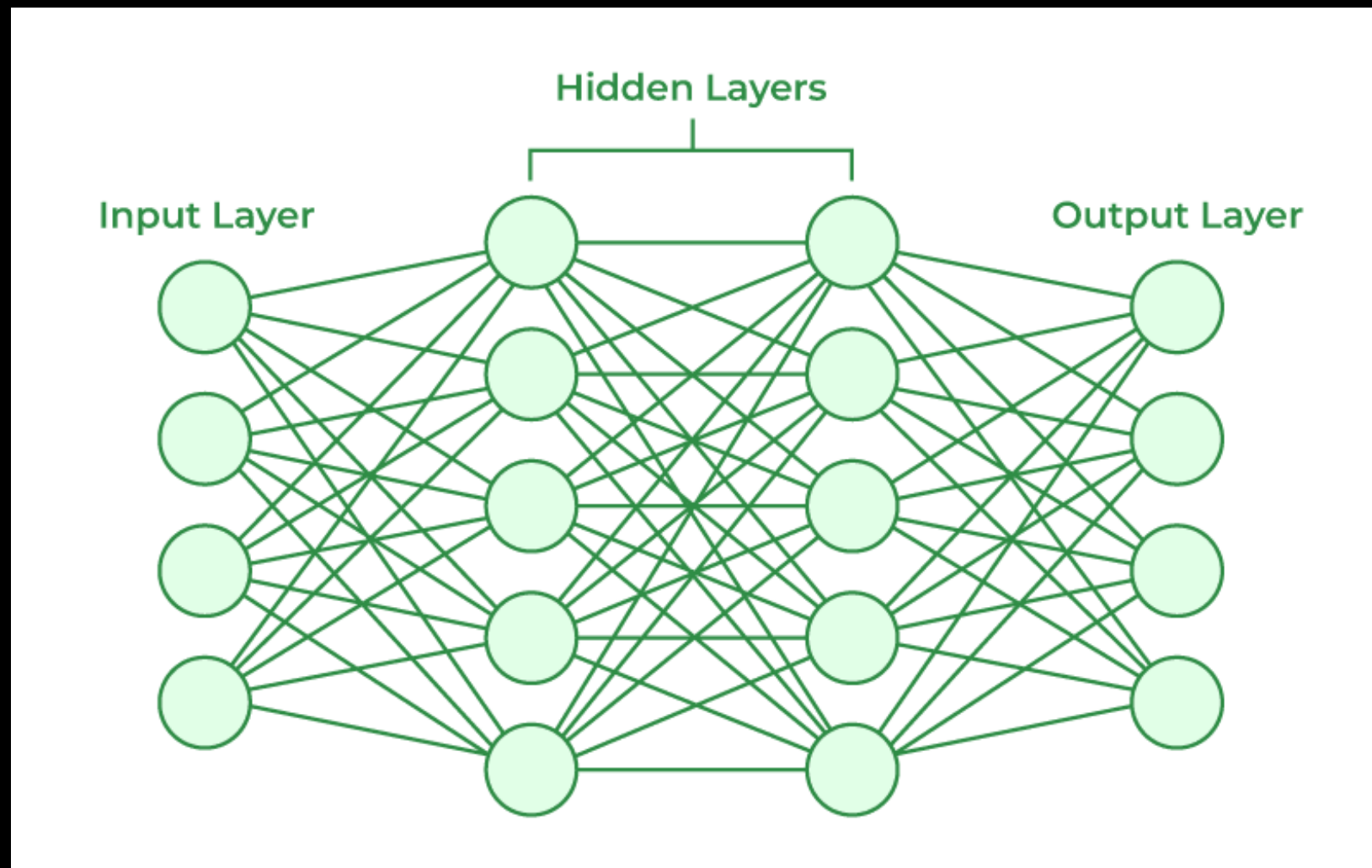
# Current Reflection Models for Data Analysis

$$F_{\text{obs}}(\nu_{\text{obs}}) = \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_1(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,1}) dg^* dr_e \\ + \frac{1}{D^2} \int_{r_{\text{ISCO}}}^{\infty} \int_0^1 \frac{\pi r_e g^2}{\sqrt{g^*(1-g^*)}} f_2(g^*, r_e, i) I_e(\nu_e, r_e, \vartheta_{e,2}) dg^* dr_e ,$$



# Neural Network

- Interconnected group of nodes that can be "trained" to make predictions



# Neural Network

- Model parameters  $\rightarrow$  Spectrum
- Spectrum  $\rightarrow$  Model parameters
- Surrogate model (see gravitational waveforms)

# Conclusions



# Conclusions

- Current XRS measurements of black holes can be accurate and precise, but only if we carefully select the sources and observations
- The analysis of observations from the next generation of X-ray missions (eXTP, Athena, HEX-P, etc.) will necessarily require more sophisticated synthetic spectra than those available today
- New generation of reflection models (neural networks?)

**Thank You!**